

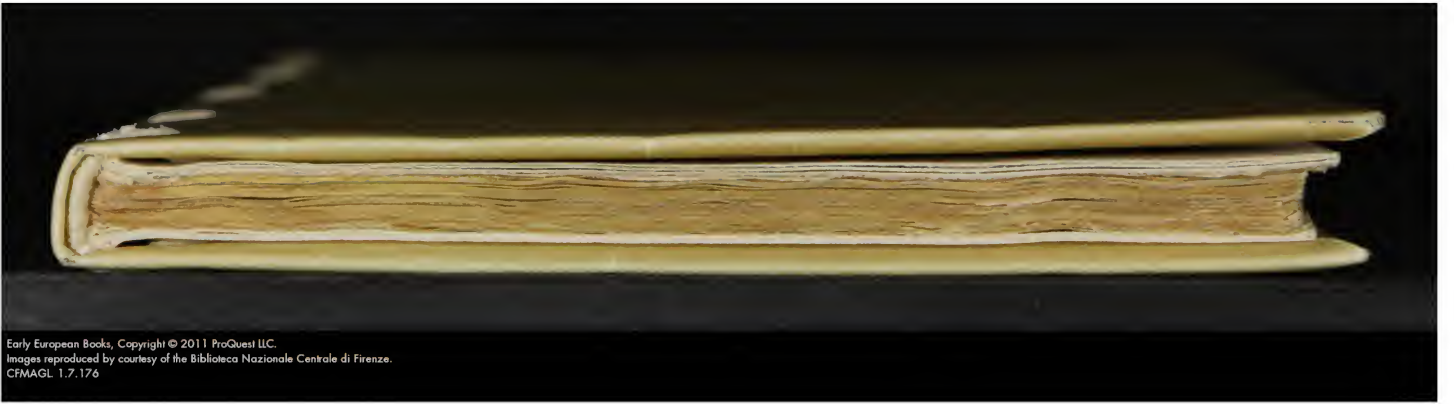
Early European Books. Copyright © 2011 ProQuest LLC.  
Images reproduced by courtesy of the Biblioteca Nazionale Centrale di Firenze.  
CFMAGL. 1.7.176



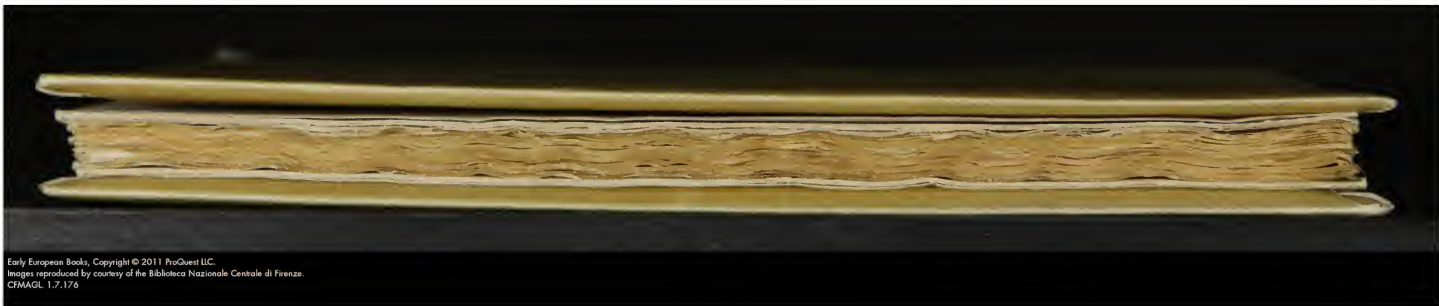


Early European Books, Copyright © 2011 ProQuest LLC.  
Images reproduced by courtesy of the Biblioteca Nazionale Centrale di Firenze.  
CFMAGL 1.7.176

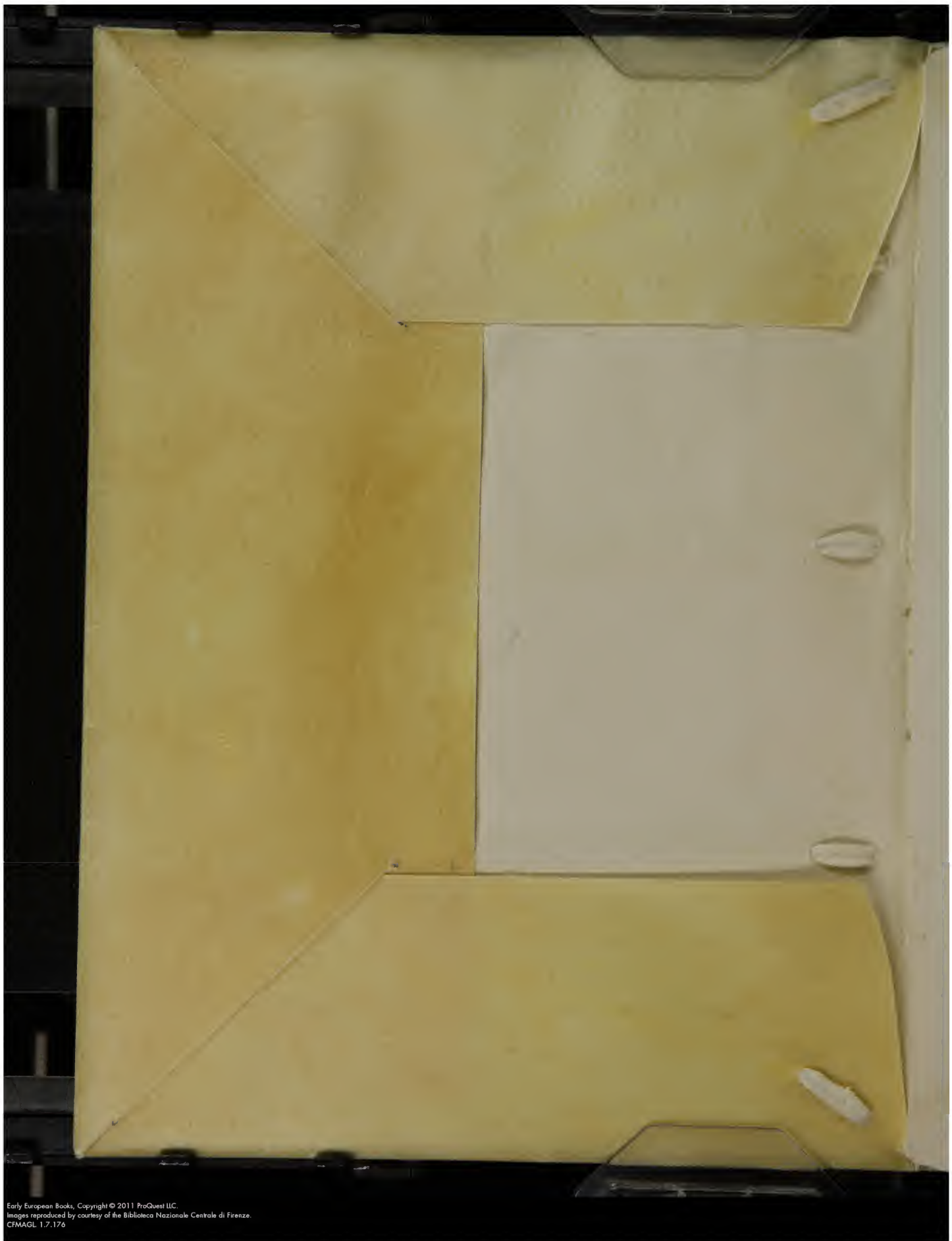


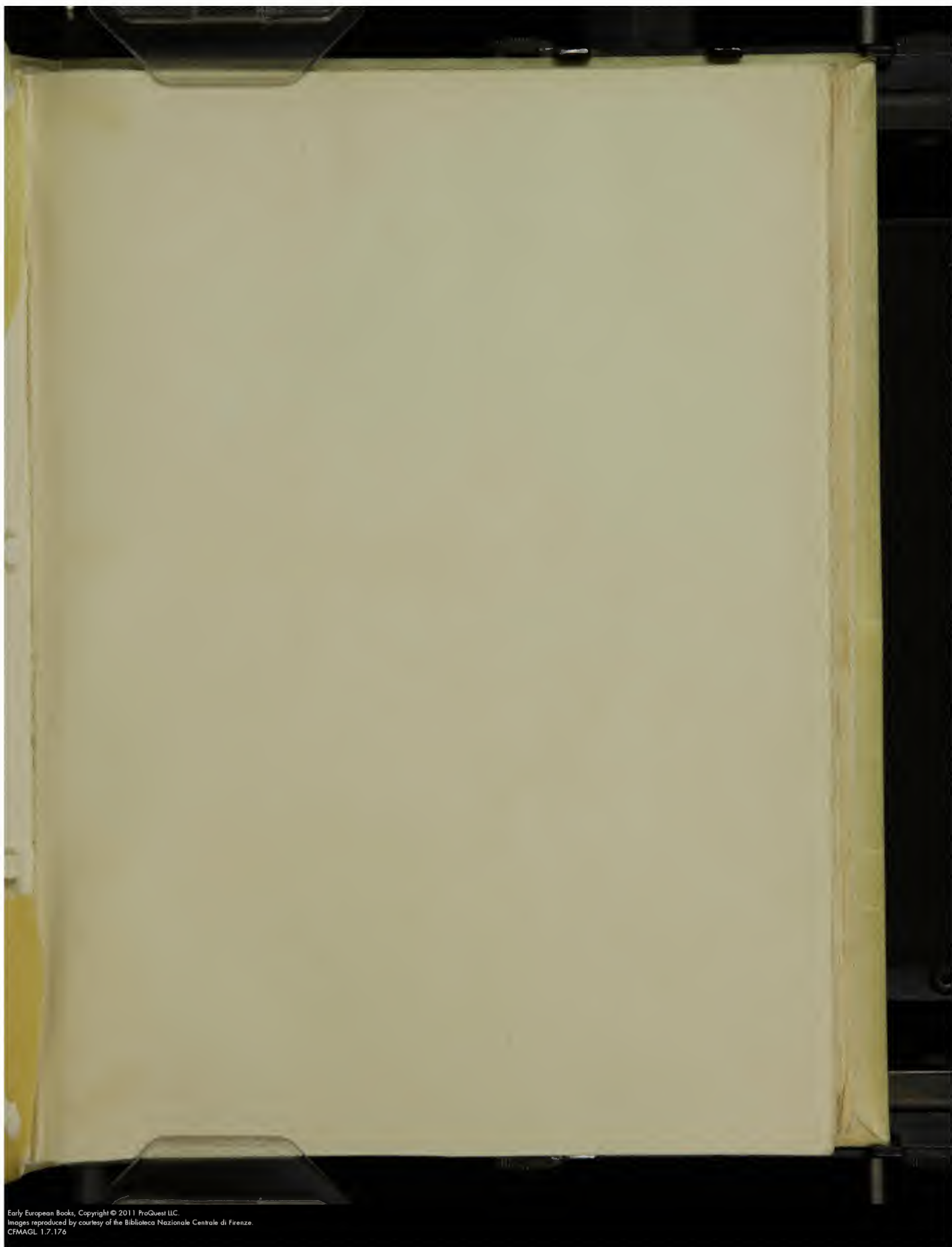


Early European Books, Copyright © 2011 ProQuest LLC.  
Images reproduced by courtesy of the Biblioteca Nazionale Centrale di Firenze.  
CFMAGL 1.7.176



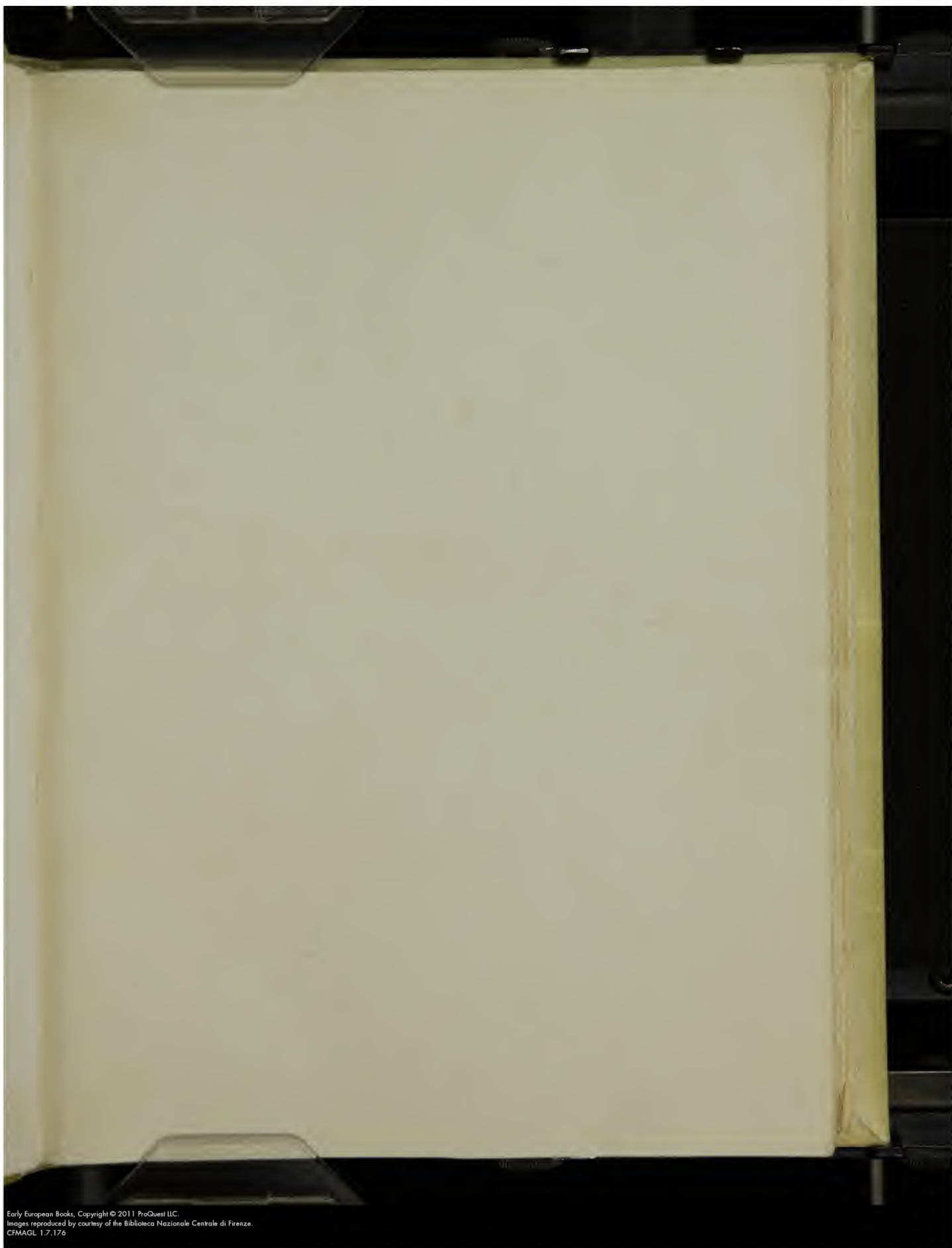
Early European Books. Copyright © 2011 ProQuest LLC.  
Images reproduced by courtesy of the Biblioteca Nazionale Centrale di Firenze.  
CFMAGL. 1.7.176

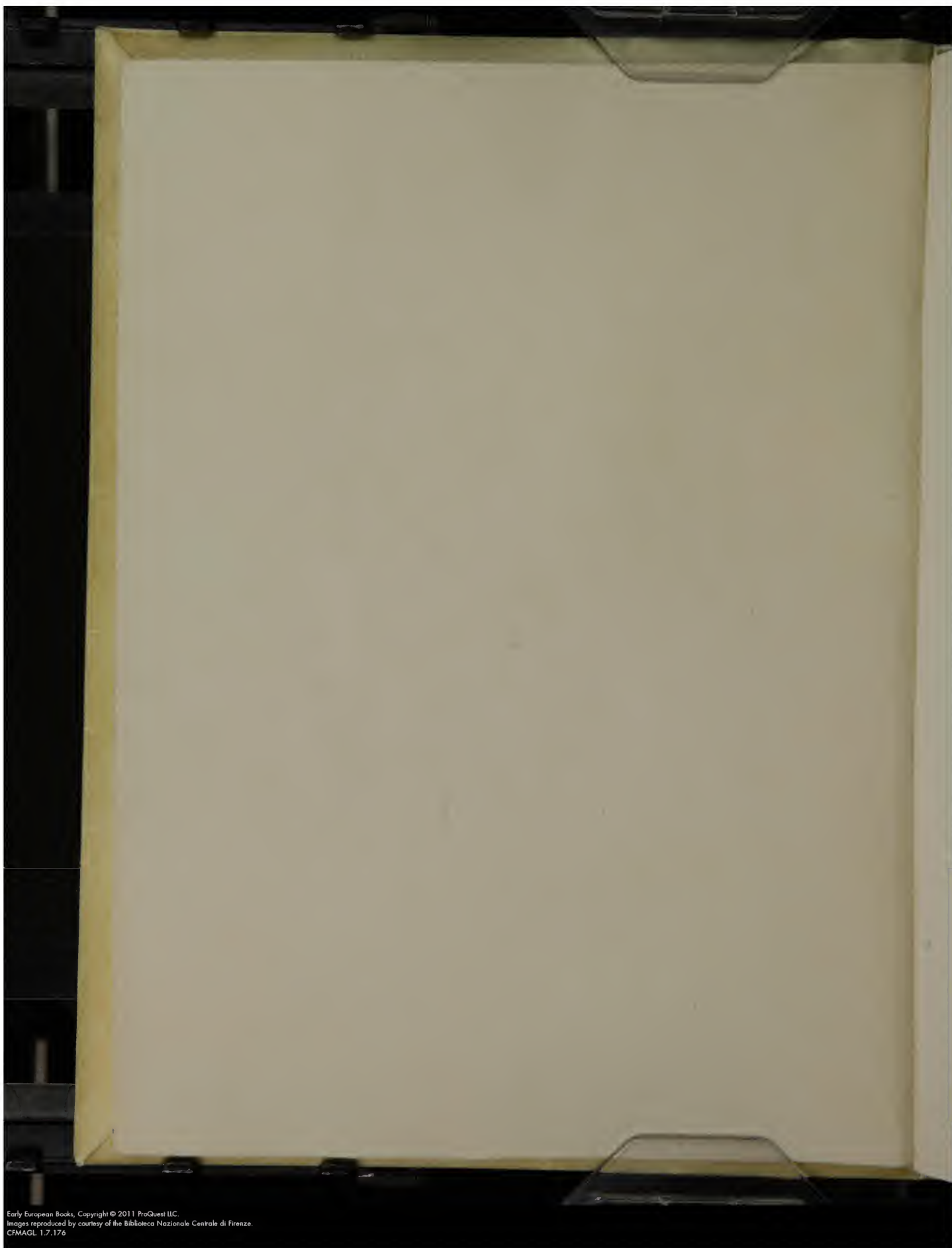


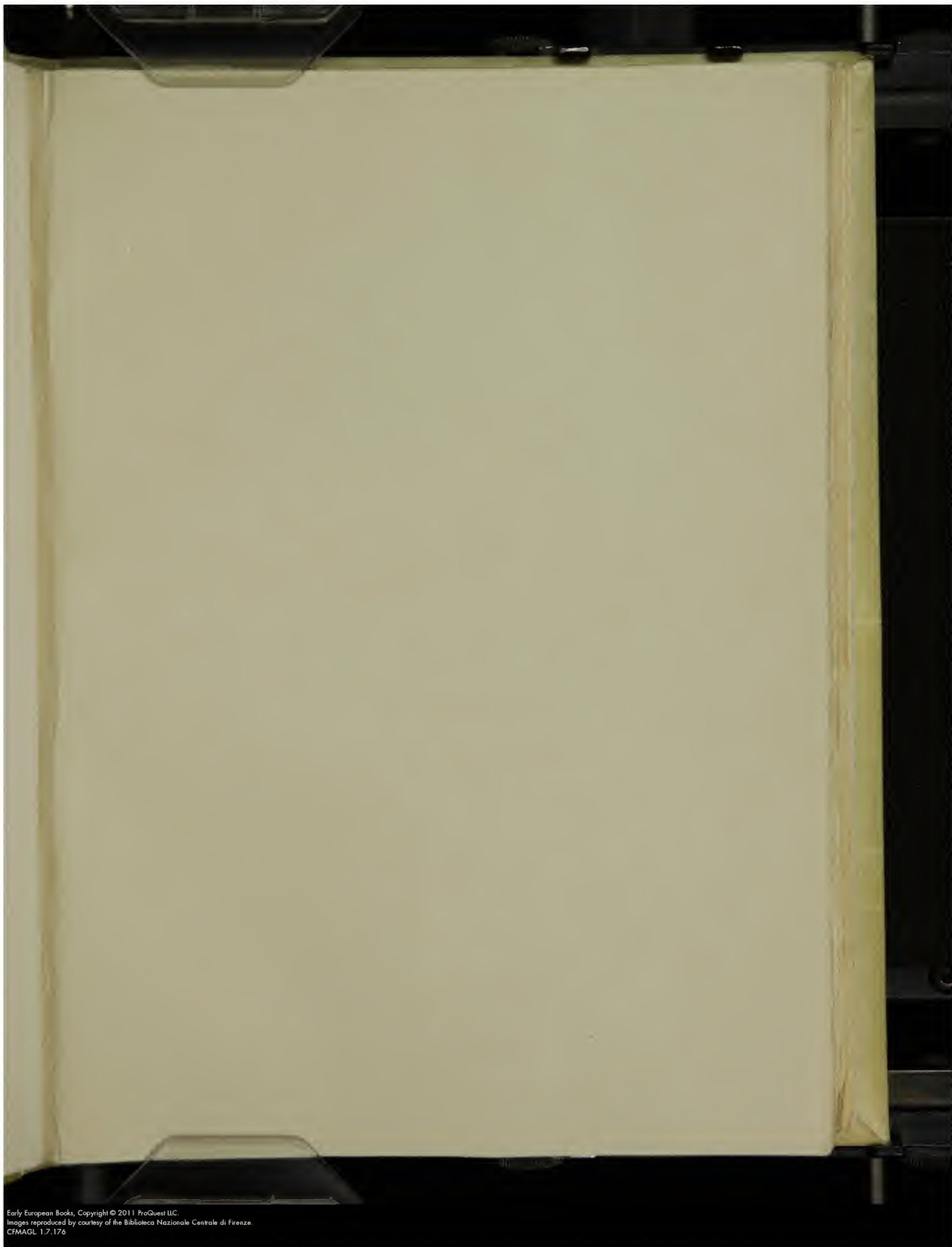


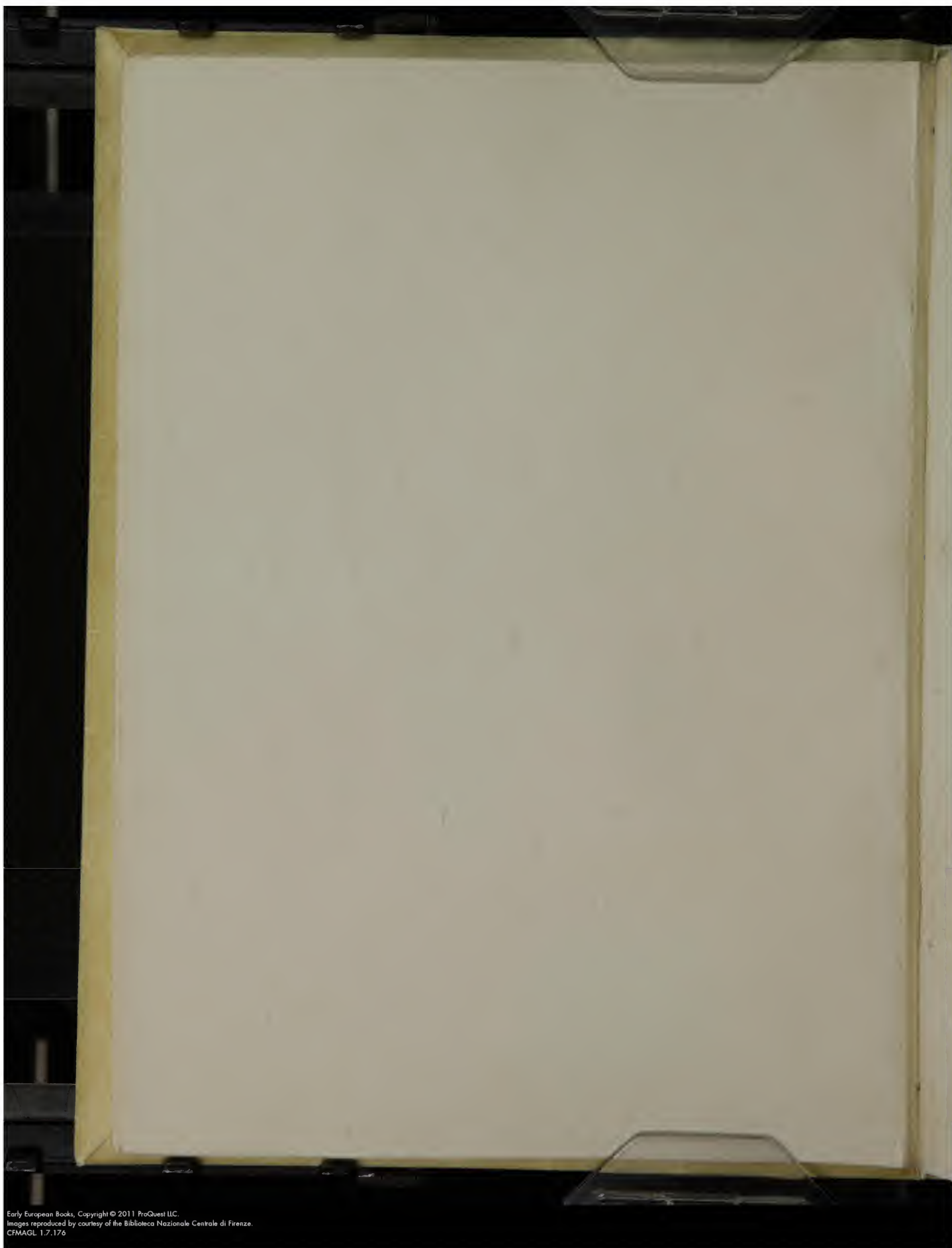
1.7.176







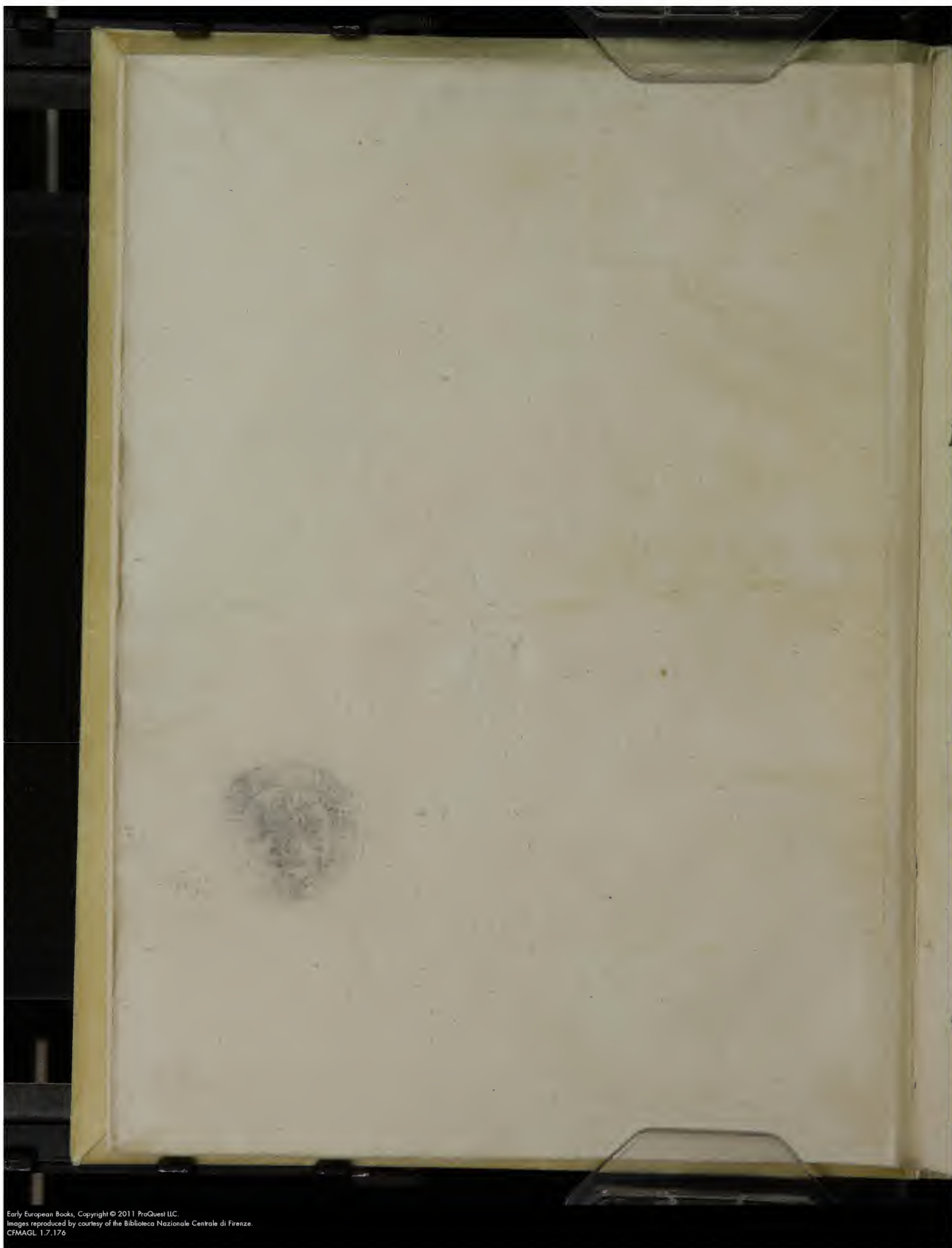




XI

PORT. Eleu.  
Curvib.





IO. BAPTISTAE PORTAE  
NEAPOLITANI  
ELEMENTORVM CVRVILINEORVM  
LIBRI TRES.

In quibus altera Geometriæ parte restituta, agitur de  
CIRCVLI QVADRATVRA.

Ad Illustrissimum Principem ac D.  
D. FEDERICVM CAESIVM  
MONTIS CAELII MARCHION. II. &c.  
BARONEM ROMANVM.



ROMAE,  
Apud Bartholomæum Zannettum. M. DC. X.

---

SVPERIORVM PERMISSV.



IO. BAPTISTA PORTA  
NEAPOLITANI  
Imprimatur si videbitur R. P. M. Sac. Pal. Apost.  
Cæsar Fidelis Vicefg.

**L**ibri tres Elementorum Curuilineorum Perillustres & Excel-  
lentissimi D. Ioannis Baptista Porta Neapolitani, ex ordine  
Reuerendissimi P. Magistri F. Ludouici Ystella Sacri Palatij Apo-  
stolici Magistri perlegi, eosque cum nihil fidei, vel moribus ad-  
uersum continere inuenerim typis dignos existimavi. Roma Die  
20. Iulij 1610.

Antonius Butius Fauentinus Ciuis Romanus  
Philosophia & Medicina Doctor.

Imprimatur. Fr. Damianus à Fonseca magister, & Socius Re-  
uerendissimi P. Magistri Fr. Ludouici Ystella, sacri Palatij  
Apostolici Magistri, Ordinis Prædicatorum.

In Clarissimum ac Doctissimum Virum  
IO. BAPTIST. PORTAM NEAP. LYN.  
& in librum de Circuli Quadrato.

*Ioannis Demisiani Cephalleniensis D. Philosophi  
ac Theologi.*

**Η**ΜΟΣ εἰς ἐπίτασι πολύχρσα Δαίδαλα ΠΟΡΤΗΣ  
Φαίνε, ποῖς θαλίθρ γαῖα πυκαζομένη.  
Οὐαπ θαμβαίνοντα βιαρκεί μῦθεν ἀκέρ,  
Καὶ σφιτέρης γαίης φέγγειν ἀγλαίης.  
Λξυγίτε βάζοντες ἀπείριτα θαύματα πόντου,  
Σιγαλήη πόντος νηπιμή γελᾶ.  
Ηέρες ἀγλήεντος ὅταν χύσιν αὐτῆς ἐσώσῃ,  
Ισὺν ἀπασράπῃ δώμασιν ἐρανίων.  
Αἰδέρες ἀσροχίτωνος ἀταρξία νῶτα πράσῃ,  
Καὶ Πόλος ἡριμίων ἐδ' ἐπέσσει δίδ.  
Κύκλα δὲ, καὶ Τεξάγωνα, Τείγωνά τε, Πείσματ', Κώνες,  
Καὶ γραμμὰς μίξῃ, κέντρα τε περαμίδων.  
Οἷα τε πρὸς Νείλου περὶ χῆς μπιόσατο τίχη  
Λήϊα δαιξέων Ηερίης ναίτης.  
Αὐτῇινω Σεφίη πολυμήχανα δήνεα τίς,  
Τῶν περὶ Νείλου ἐρυκακέφ.  
Εδρακε γὰρ Τεξάγωνα πάρος πολεμήϊα Κύκλοις,  
Ορκία σκευδοίης, καὶ φιλήης ταιμείν.  
Εδρακε, καὶ θάμβησεν ὅπερ χρόνος ὅτ' ἐφαίνεν  
Ημελλε, ζαθέων ἡὺ τέκος περαπίδων.  
Αλλοι γὰρ Κεγνίδη δώσω πολυῖδ' ἄμωνα ΠΟΡΤΗΝ,  
Ηὶ ἡμῖν ἄλλω Παρθινόπῳ ὁπάσῃς.



FRANCISCI STELLVTI

MY FABRIANE NSIS. IO

LYNCAE. Aprilis 30

*Vidimus innumeras mutantem Protea formas,*

*Credite, nam veri Nuncia Fama canit.*

*Tortilis en Orbis species se vertit in omnes,*

*Et QVADRV M. teretes efficit arte rotas.*

*Dicite Pierides quo tandem munere factum?*

*Aut nostro, aut PORTÆ. visq. laborq. pares.*

ILL.



ILL<sup>MO</sup> PRINCIPI AC D. D.  
FEDERICO CAESIO  
MONTIS CAELII  
MARCHIONI II.

*Io. Baptista Porta Neapolitanus. S.*



ERTAMVS inter nos, Illustris-  
sime Vir, tu beneficijs, ego officijs,  
quibus equo animo vel vincar abs  
te, vel, si fieri posset, vincam te.  
Et sanè grauis ista contentio nul-  
lum vnquam finem habitura vi-  
derur. Summis me ornas laudibus,  
meos libellos plausu, nedum honore prosequeris, &  
quod caput est, iacentes aliquando, ac mox improbo-  
rum impetu proterendos, erigis & defendis; quæ qui-  
dem merita ita in memoria insederunt mea, vt mei ip-  
sus potius, quam illorū erga me magnitudinis obliuio  
capiat. Ego verò si titulos percensere velim, quibus tuū  
animum virtus cohonestauit, splendorem domus, quam  
„ Bellipotens illustrat Auus, Tu fulcis, & ornas.  
aliaq. ornamenta, quibus te natura mirificè cumula-  
uit; & vires, & vita me deficeret. quid? ipsam Inui-  
diam ad maxima quæque, ac pulcherrima labefactan-  
da natam, virtute superasti.

„Est

„ Est aliquod meriti spatium, quod nulla furentis  
„ Inuidiæ mensura capit.

Sed non est animus in præsentia laudes enumerare tuas.  
maioris id molis est. leuiter, at amanter tetigisse satis;  
neque enim qui Cœlestium Orbium ornatum in parua  
describunt tabella, de illorum pulchritudine quicquam  
demunt; parua, vt ita dicam, sed concinna magnitudo.  
Quid igitur mirum si certos fines, terminosq. huic sua-  
uissimæ concertationi non constituo? Non patitur mea  
in te obseruantia Victoriâ. Tu, quæ tua est magnani-  
mitas, cedere nescis. Esto lis sub te Iudice. Tu te vince  
„ Inq. animis hominum pompa meliore triumphâ.  
meum certè quidem tibi deuinxisti, ac deuicisti. Non  
excitabo testes ex monumentis; quæ in manus perue-  
nerunt Sapientum. Sit hic liber tuo insignitus nomi-  
ne, amoris, ac venerationis in te meæ pignus semper-  
num. Circulum quadrare conaturum scilicet aggressus  
in eruditorum identidem commemoratam comi-  
tijs, in Philosophorum agitatam scholis, in Mathema-  
ticorum iactatam iudicijs. Multos in hoc Theoremate  
me labores exantlassè, curas, & cogitationes euigilassè  
meas; ac pertinaci industria desudassè, non inficias iue-  
rim. An verò modum quadrandi Circuli inuenerim,  
sicq. præmium, & fructuum meorum cœperim laborû,  
non facile statuerim. Id saltem affectus mihi videor.  
Latissimum aperuisse campum ad meliora vel inue-  
stiganda, vel inuenienda. Verecundè tamen dixerim,  
plurima nos excogitassè, multa in disquisitionem vo-  
casse,



11  
casſe, ſuiſq. examinafſe ponderibus, quæ nemo uſque  
in hodiernum diem odoratus quidem eſt. Immo, ut  
id quod ſentio, aperiã, opus magnis uiris tentatum, ac  
tandem deſperatum, aut inchoauimus, aut perfecimus.  
nihil tamen in tanto, ac tali negotio pro certo affirma-  
rim, te, non aſſentiente, tuæ enim *παλ λάδος υπό πρεſβς  
ἐν τῇ ἀζον βεβη*. tuo iudicio, ac patrocinio fultus, non  
morabor *τῷ Γεφνεῖς*. Tenes, opinor, memoria, in-  
comparabilis uir, Ephēſiorum factum. Illi dum ho-  
ſtili vexarentur bello, de rei euentu conſuluerunt Ora-  
culum. datum reſponſum, ſi Rempubicam ſartam te-  
ctam cuperent, ad Tutelaris Numinis Templum Urbē  
alligarent; quo peracto, hoſtes in fugam uerterunt,  
Ephēſumq. obſidione, ac metu liberarunt. Multi iam  
cogitant noſtra obſidere inuenta, machinas admouent,  
ac penē labefactant: ſed meus Apollo dudum me com-  
moneſecit, ut me meaq. tui Genij uinculis obſtricta,  
aduerſariorum impetus reprimam, ac frangam. Tuere  
igitur, Heros, litterarum, ac litteratorum Cenſor, quæ  
tibi dicata ſunt, eo vultu, quo intuentium allicis ani-  
mos. Habes à Philoſophia non minora clementiæ,  
quam iudicij præſidia, ut illa nouos hoſce foueas cona-  
tus, hoc ut defendas. Vale, tecumq. creſcat tuæ Gen-  
tis ſpes, Patriæ columen, litterarum decus, meæ Nea-  
poleos amores, Italiæ gloria. Kal. Iulij M. DC. X.

A D

# AD LECTOREM

## PRAEFATIO.



**N**ON immeritò, Candide Lector, admirari satis non possumus de viris quibusdam omni doctrinae genere cumulatis, qui, cum mathematicas tractationes sibi assumpserint, atque in ijs cum laude versati, sint, de illa parte, quæ curvas complectitur lineas, nihil ferè commentati, aut meditati sint. In quadrando quidem certè Circulo (re scilicet æquè decantata, atque ardua) plerique ingeniosi viri desudarunt, & elaborarunt rectè ne, an secus, ipsi viderint. Ego qui noui aliquid moliri, non aliorum labores veluti fucus surripere studeo, eandem quidem subiui aleam. Sed ut legitime & expeditius id præstarem, multa ex Euclideis elementis ad propositum argumentum transtuli, ac plurimas confeci demonstrationes, ex quibus, aliquas, quæ ad rem facere videntur se legi, easq. vti curvilinearum figurarum elementa proposui. Hinc ad perdifficile Theorema de quadrando Circulo, progressus sum. quid vero effecerim in re multis circumfusa tenebris, & in quâ summorum virorum ingenia errare potius, quam bærere visa sunt, aliorum esto iudicium. si perfectionem non sum omnino affecutus, conatus certè, & adumbratio tanti Theorematis laudandus.



I  
IO. BAPT. PORTÆ  
NEAPOLITANI  
ELEMENTORVM CURVILINEORVM  
Liber Primus.

DEFINITIONES.

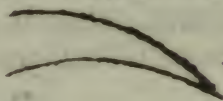
PRIMA.

**L**INEA curua est, quæ inter sua nõ æquè fluit puncta, sed facto sinu flectitur.

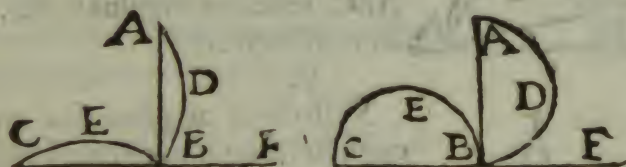


II.

Angulus flexilineus est flexarum linearum retusio suo nuru sibi coincidentium.



III.



Angulus flexilineus rectus, qui rectilineo respondet.

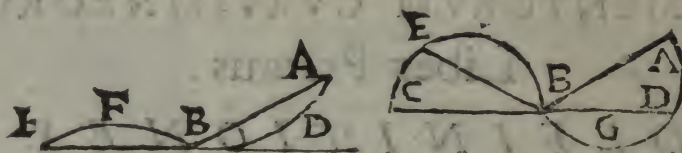
Exempli causa sit A.B. insidens linea iacens FBC. utrobique sibi æquales constituens angulos ABF, ABC. sitq. AB. ipsi B. C. æqualis & ipsi AB. hemicyclium circumscribatur ADB, vel circuli portio, & ipsi BC. alter B. E C. vel æqualis circuli portio. Cyclogoni ergo DBA. CBE. sunt æquales, & quanto angulus AD.B.F. maior est recto ipso contingentia angulo DBF. tanto ABE. superat ipsum ABC. altero contingentia angulo ABE.

A totus



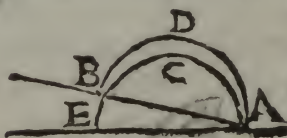
totus igitur  $ADBE C$ . toti  $ABC$ . recto æqualis, vt probauit Proclus in Eucl.

## III.



Obtusus curuilineus, qui obtuso rectilineo fit quando à recto resupinata in maiorem angulum abit.

Eodemq. modo angulū  $ADB$ . flexilineum, rectilineo  $ABE$ , esse æquale flexilineus angulus  $FBE$  est æqualis flexilineo  $DBG$ . nam æquales sunt circulorum portiones, si angulum  $DBG$ . abstuleris, & reposueris supra  $EB$ . erit rectilineus  $DBE$ . æqualis flexilineo  $DGBEE$ .



Sic etiā semicirculus  $ADB$ . æqualis est  $ACE$ , dematur portio communis  $ABC$ . remanet angulus  $CAD$ . æqualis rectilineo  $BAE$ .

## V.



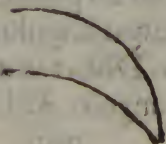
Xystroides angulus siue concauus quando vtrarumq. circumferentiarum caua extra fuerint, & intus se respiciens conuexitatibus suis.

## VI.



Contra conuexus angulus quando circumferentiarum conuexa vtrinq. extra fuerint, & inter se suis finibus aspexerint.

## VII.



Angulus *lunaris*. siue lunaris, qui ex caua conuexavè circumferentia fuerit, vt conuexum vnus alterius conuexitatem aspiciat.

Cyf-

## VIII.

Cyssoides Angulus ex hederæ folijs nomen indeptum ex gibbosis, cauisq. lineis constat ad punctum vnum conuenientibus, vndatim contra se discurrentibus veluti Vndulatus.



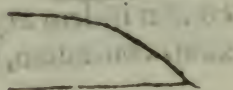
## IX.

Mixtus angulus, qui ex rectis circulosiq. lineis componitur.



## X.

Cyclogonus, qui à Caua, & recta circuli circumferentia constat.



## XI.

Κερατοειδής. siue in cornua falcatus, quando rectæ opponitur conuexa nostri contingentiæ vocant.



## XII.

Figura vel angulosa, vel agonia, agonia- rum figurarum circulus princeps, lineæ partem, quæ ambitiosè circumuoluitur, & aream obambit concauum dicimus, quæ extorsum inuehitur conuexum.



## XIII.

Sphærois siue Ellipsis ex ambienti lineâ in se recursa describitur vnius duæ diametri, longitudinis vna longior, latitudinis altera ad rectum in medio se secantes.



## XIIII.

Vertex. siue corona est duorum circulorum concentricorum circumcursus.



## XV.

Angulosarum figurarum metrisus siue

A

2

lu-



lunula prior, estq. in easdem partes  
caua habentibus comprehensa cir-  
cumferentijs figura.

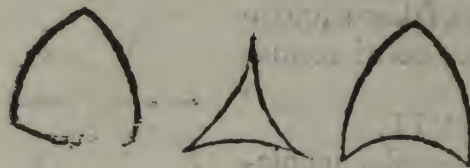


XVI.



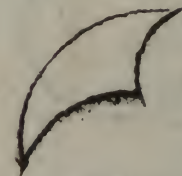
Trilaterarum  
figurarum flexi-  
linearum trian-  
gulum primum  
est, quod tribus  
constat ijsdem æqualibus circumferentijs circuli, idq. conte-  
xum, concavum, vel mixtum.

XVII.



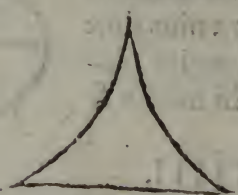
Isoscele triangulum  
curvilineum, quod dua-  
bus tantum æqualibus cir-  
culi circumferentijs con-  
tinetur, idq. etiam con-  
vexum, vel concavum, vel mixtum.

XVIII.



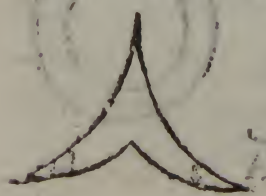
Scalenum flexilineum est, quod tribus in-  
æqualibus circuli circumferentijs clauditur,  
ijsq. cauis convexis, & mixtis.

XIX.



Semicurvilinea trian-  
gula sunt, quæ ex rectis,  
curvisq. circumferentijs  
continentur.

XX.



Tricuspdatum triangulum, siue acio  
idea quadrilaterum est triangulum  
quod tres habet acutos angulos.

Inter

## XXI.

Inter triangulares figuras *παρακοιδή*. Figura est, quæ securis vel bipennis formā habet.

Eius Theocritus meminit. Nicandri Scholiastes sutorium scalprum. *Τὰ κυκλοπεῖρη σιδήρεα, οἷς οἱ σφυτοτόμοι τέμνουσιν καὶ ζεύουσιν τὰ δερμάτα*. Idest circularia ferramenta quibus pelles incidunt, & deradunt.



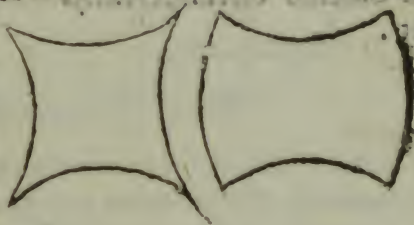
## XXII.

Arbilones ex tribus circumferentijs comprehensi; Horum meminit Pappus spatium illud inter circumferentias interiectum *ἀρβιλον* vocans.

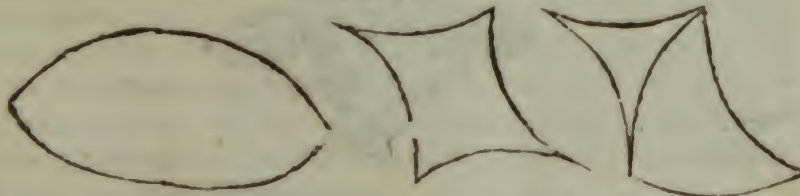


## XXIII.

Quadrilaterarum quidam figurarum curvilinearum quadratum quidem flexilineū est, quod rectis angulis, & æqualibus circumferentijs describetur.



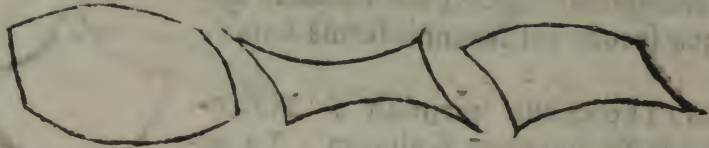
## XXIIII.



Rhombus flexilinea æquilatera quidem, sed non rectangulara, aduersos tamen angulos æquales habet, eorumq. aliquos concauos, conuexos, & mixtos.

Rhom-

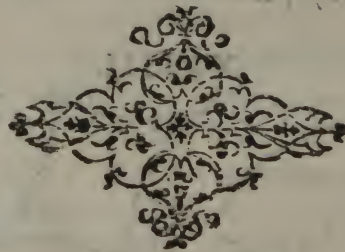
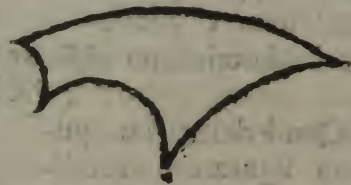




Rhomboides vero neutrum horum habet neque laterum, neque angulorum æqualitatem, sed contrarias circumferentias, & angulos æquales habet similiter etiam concavus, convexus, & mixtus.

## XXVI.

Trapezoides curvilineum, quod quatuor inæqualia latera ex diuersis circumferentijs habet.



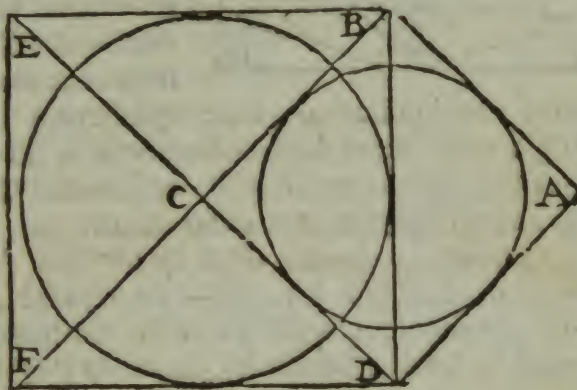


## PROBL. I. PROP. I.

Datum circulum duplare.

**S**IT datus circulus  $ABCD$ . cuius oportet duplum inuestigare. Describatur quadratum per 7. 4. Eucl. & sit  $ABCD$ . ducto Diagonio  $BD$ . secundum datum  $BD$ .

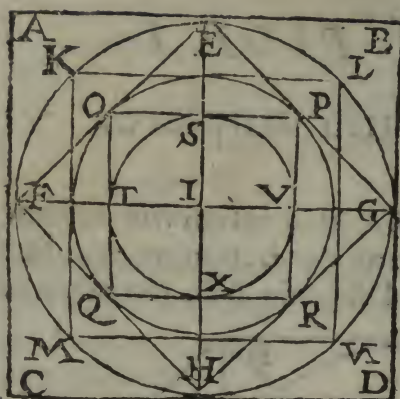
describatur quadratum per 48. 1. Euclidis, & sit  $BEDF$ . cui circulus inscribatur per 6. 4. dico circulum  $BDFE$ . esse dati duplum. Hanc



constructionem demonstratione fulciendam rati sumus. quoniam  $BCD$ . rectus est angulus proinde cum quadrata lateris  $BC$ .  $CD$ . æqualia sint quadrato ex  $BD$ . ex 47. 1. ergo quadratum ex  $BD$ . duplum quadrati  $ABCD$ . sed ex  $BD$ . descriptum quadratum est  $DBFE$ , ergo quadratum  $BDEF$ . duplum ipsius  $ABCD$ . sed circulus ad circulum eandem rationem habet, quam quadratum inscriptum, aut circumscriptum, ut ex Euclideâ demonstratione ratum est duodecimi elementorum secunda, ergo circulum  $ABCD$ . duplaui-  
mus per circulum  $BEFD$ .

Plato ita quadratum duplat ut à Vitruuio annotatur. Dimidium quadrati  $BD C$ . est quarta pars quadrati  $BEF$ . ergo quadratum  $BEDF$ . duplum est  $ABCD$ .

Possu-



oportet conduplicare, huic quadratum circumstruemus  $OPQR$ . cuius latera duabus diametris se ad centrum  $I$ , decussantibus bipartiemur, & circa quadratum  $OPQR$ . circulus alter designetur mox aliud quadratum.  $KLMN$ . & alter circulus, ac demum aliud quadratum  $ABCD$ . quod postremum circulum  $KLMN$ . intercludat. His perstruētis aio aream inter circuli  $KLMN$ . finitionem concludam proximè septientis arctioris sui circuli  $OPQR$ . duplam esse, ut laxior postremi area eius, qui minimum intercludit quadrupla sit, & sic in infinitum duplicare possumus cuius veritas hac demonstratione repræsentabitur. Quoniam linea  $AB$ . bifariam diuisa est in  $E$ , quadratum  $ABCD$ . quadruplum est ipsius  $AE$ , & sic in quatuor quadrata æqualia  $AI$ ,  $EG$ ,  $FH$ ,  $ID$ . & hæc à quatuor diagonijs bifariam diuisa sunt  $EF$ ,  $FH$ ,  $HG$ ,  $GE$ , quatuor igitur triangula extrinseca,  $FAE$ ,  $EBG$ ,  $GDH$ ,  $HCF$ . quatuor interioribus æqualia, sunt; ergo totum quadratum  $ABCD$ . quadrati  $EFGH$ . duplum erit, eademq. ratione quadratum  $EFGH$ . ipsius  $OPQR$ . duplum erit, & primum  $ABCD$ . huius quadruplum.

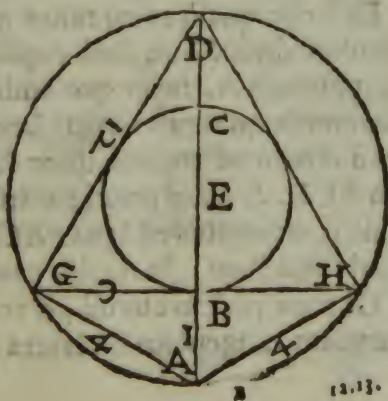
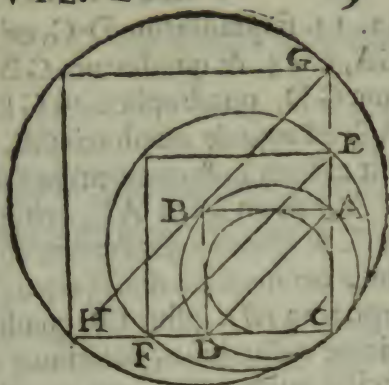
Anni-



Annitemur etiam per quadrata dupla ambientia idem rimari, & absolvere. Esto datus circulus  $ABCD$ , cui quadratum  $ABCD$ . circumstruimus: mox ab oppositis angulis ducto diagonio  $AD$  & à puncto  $C$ , superne versus  $A$ , signa lineam eiusdem longitudinis ipsius  $AD$ , & sit  $CE$ , & ex parte inferiori sit  $CF$ , mox trahè diagonium  $EF$ ; & iterum quanta  $EF$ , figura in linea  $CG$ , & inferne linea  $CH$ , & id toties repetendum quoad satis videbitur. Sic quadratum ex  $GH$ , duplum est quadrati  $EF$ , & quadratum  $EF$ , duplum  $AD$ , &  $AD$ , duplum  $AC$ . Sed quod exprimit figura, demonstramus. Quoniam quadratum  $AD$ , est æquale quadratis  $AC$ ,  $CD$ , &  $AC$ ,  $CD$ , latera æqualia sunt, ergo quadratum ex  $A$ , duplum est quadrati  $AC$ , sed  $AD$ , est æquale  $EC$ , ergo quadratum  $EC$ , est duplum  $AC$ . Sed quia  $EF$ , est æquale duobus quadratis  $EC$ ,  $CF$ , &  $EC$ ,  $CF$ , æqualia sunt, ergo quadratum ex  $EF$ , duplum est  $EC$ . Eodem modo  $GH$ , duplum ipsius  $AC$ , & si idem varijs modis assequi posset, tanquam suffecturos reliquos censuimus missos facere.

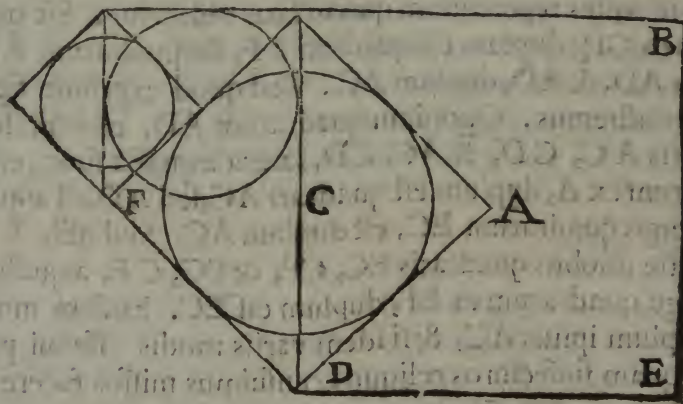
Libet non prætermittere, alium quadruplandi modum.

Sic circulus  $BC$ , quem intendimus quadruplare circa quem æquilaterum triangulum per tertiam quarti describamus, & circa illud alium circulum per quintam eiusdem quem quadruplum pronunciamus. Quoniam  $DG$ , tripla est ipsius  $GA$ . ex





12. 13. si quadratum  $DG$ , erit duodecim partium talium.  $GA$ , erit 4. & quadratum  $GB$ , erit talium 3. nam quadratum  $GD$ , quadruplum est  $GB$ , suæ dimidiæ, sed quadratum  $AG$ , est æquale quadratis  $GB$ ,  $BA$ , igitur si quadratum  $GA$ , erit talium 4. & quadratum  $GB$ , talium 3. erit quadratum  $BA$ . talium 1, sed  $AE$ , erit quatuor, quoniam est æqualis  $AG$ . & quando quadratum totum 4. est, & sui pars 1. erit linea per medium diuisa ergo  $AB$ . ipsius  $AE$ . dimidium erit, ergo tota  $AD$ . ipsius  $EB$ . quadrupla est. Si vero circulum diuidere voluerimus, poterimus conuersa vti operatione; Et si facilia quidem sint, quo tyrones iuuenus alium modum apponere non pigebit.



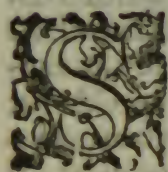
Describe quadratum tantæ quantitatis quantæ duplarem circulum diuidendum fieri cupis, & sit  $ABCDE$ . cuius medio fige punctum  $A$ . super quo ambitiosa linea circumducatur, quæ omnia quadrati tangat latera, deinde annecte literas rectas à centro ad angulos duos  $AC$ .  $CD$ . & constitue triangulum  $ACD$ . & aliud priori par triangulum constitue cuius angulus  $F$ . erit rectus est igitur  $ACDF$ . secundum quadratum primi dimidium. In medio puncto huius diagonij  $CD$ , qui sit  $G$ . pone pedem circini, & reliquo vago describe circumferentiam tangentem suis latera quadrati  $ACDF$ . & hoc modo

ELEM. CVR VIL LIB. I.

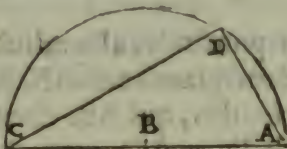
do in infinitum poteris circulos dimidiare. Demonstratio ex superiori pendet.

Datum circulum triplicem quintupli-  
cem, & septuplicem reddere.

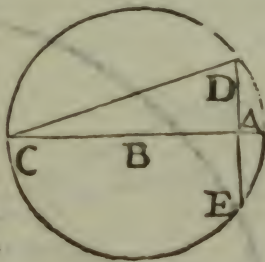
Prob. 2.



IT dati circuli diame-  
ter AB. quem volumus  
triplare elongetur AB.  
in C. & sit AB. æqualis  
BC. & fiat circulus ex  
diametro AC. & sit AB. æqualis AD. quæ in circulo loce-  
tur per primam 4. Euclid. & ducatur DC. dico circulum ex  
DC. diametro circuli ex AB. tripli esse cuius demonstratio ex  
12. 13. lib. Eucl. pend.



Si vero quintuplare voluerimus sit  
data diameter AB. circuli quintuplan-  
di. Elongetur quantum AB. & sit BC.  
circumducatur ei circulus ADC. in  
quo pentagonum æquilaterum inscri-  
batur per 9. 4. Eucl. & sit linea subten-  
dens duobus lateribus DC. pentagoni  
latus AC. dico quadratum DC. DE. si-  
mul iuncta quadrati AB. quintuplam  
esse. Demonstrationem quare ex 12. 13. Euclidis.

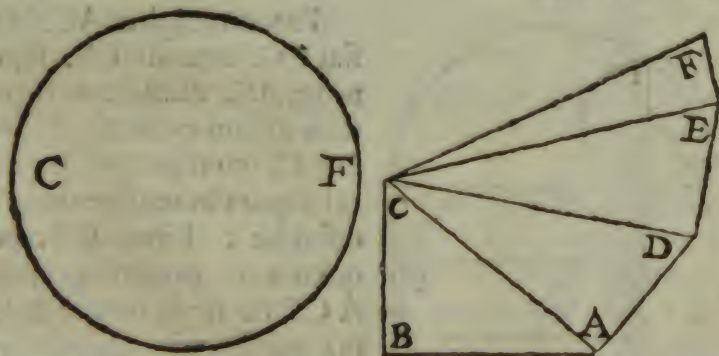
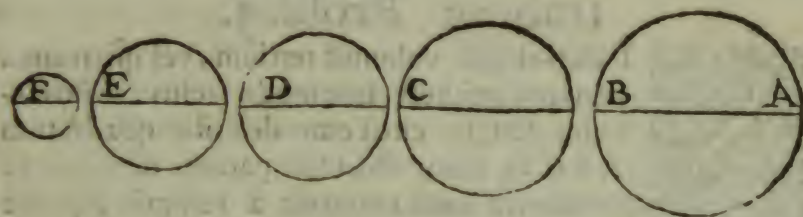


B 2 Ac





Statuatur circulus  $ABCD$ . septies multiplicandus cui circumducatur quadratum, & latus eius producemus, illudque in octo partes diuidemus, cuius principium  $D$ . finis  $E$ . mox  $DE$ . per medium diuidatur in  $F$ . positoq. circini pede in  $F$ . & alio  $DE$ . circumducatur quousq. semicirculum absoluat  $DE$ . & latus  $C. B$ . quadrati producatultra  $B$ . in continuum, rectumq. ad arcum  $DE$ , & ubi eum contingit, illic scribe literam  $G$ . & ex  $CG$ . fiat quadratum  $CGHE$ . in quo circulus inscribatur, qui continebit septies ipsum  $BACD$ . Quoniam  $CG$ . est media proportionalis inter  $EC. CD$ . igitur per 13. 6. Euclid. ut  $EC$ . prima ad tertiam  $CD$ . ita  $GH$ . quadratum secundæ ad  $BD$ . quadratum tertiæ per 20. 6. Est autem  $EC$ . per constructionem septupla ipsius  $CD$ . igitur quadratum  $HC$ : septuplum ipsius quadrati  $BD$ . quod probandum assumpsimus.

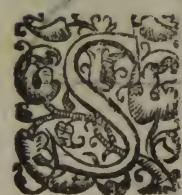


Sint positi quini circuli diuersæ capacitatis  $AB. C. D. E. F$ .  
quo-

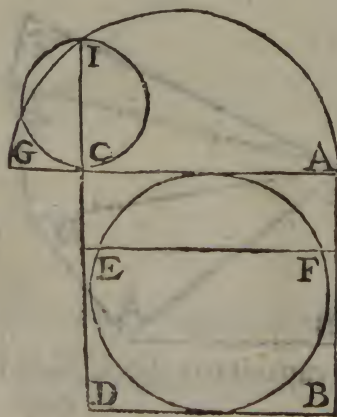


quorum quantitates volumus singulari circulo comprehendere, quod ita propemodum faciendum existimamus. Esto enim circuli diameter AB. constitutur ad rectos angulos ei BC. mox ducatur linea ab A. ad C. & hæc dimetiens potest binos circulos AB. C. Porro puncto A. lineæ AC. recta linea erigatur ad rectos angulos, quæ sit AD. & à puncto D. trahatur linea D.C. & hæc dimetiens est capiens tres circulos AB. C. D. ipsi demum CD. recta linea ad rectos erigatur DE. quarti circuli dimetiens potens quatuor circulos. Postremo ei lineæ EC. ad rectos iterum excitetur quinti circuli EF. trahaturq. per FC. dimetiens, capiens iam cunctos circulos, & hoc modo omnes licet quotquot volueris comprehendere. Demonstratio habetur ex penultima 1. libri Euclidis.

### Ex dato circulo datam partem subtrahere. Probl. 3.



I dati circuli volumus tertiam, vel quartam partem extrahere, hoc modo facito. Esto circulus ABCD. circa eum describe quadratum ABCD. cuius abscinde partem tertiam, ac transuersa linea conuenit à reliquis supernè distinguere FE.

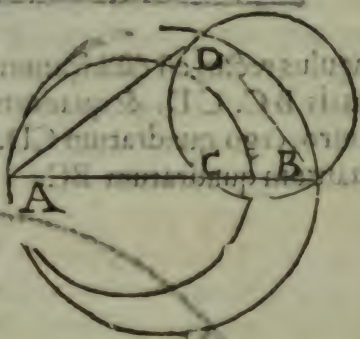


Procurrat igitur AC. in G. & fiat CG. æqualis CE. supra lineam AG. dimidium rotunditatis arcum excurrat, & linea DEC. eousque producenda erit, quo circumferentiam in I offendat. Linea CI. potest quantum parallelogrammum ACE. sic de quinta & septima parte cuius demonstratio ex vltima secūdi depēdet Eucl. Datis

Datis duobus circulis inæqualibus à  
maiori minorem subducere, &  
circulum dare reliquo  
æqualem spatio.

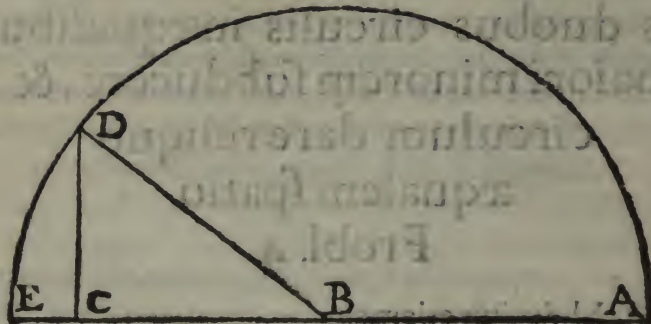
Probl. 4.

**S**ubducitur etiam cir-  
culus minor à ma-  
iori, & circulus etiā  
formari potest, qui  
vtriusque differentiam capiat.  
Esto maior circulus ABD. volo  
ab eo circulum subducere, ac  
mox alium circulum formare,  
qui lunulam ADBC. inter vtrū-  
que relictam capiat. Subducē-  
dus circulus AC. hæreat in fine  
diametri AB. in A. positoq. circini pede in A. altero ad C.  
vagum ad circumferentiam traducito, & ubi eam incidit, ibi  
locetur D. Mox ex B. ad D. transversa ducatur linea DB.  
Dico lineam DB. esse eius circuli dimetientem capientem in-  
ter AB. AC. differentiam. Quoniam trianguli ADB. angu-  
lus D. ad circumferentiam rectus est, subtensa AB. potest, ut  
AD. DB. Si igitur ex AB. subducatur AD. circulus, remanet  
alter DB. differentiam capiens vtriusq.

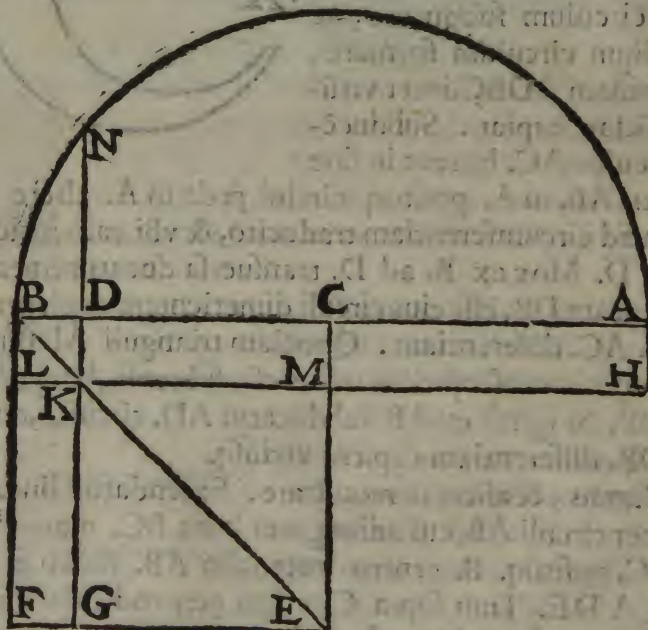


Possumus, & aliter demonstrare. Extendatur linea AB.  
diameter circuli AB, cui adiungatur linea BC. diameter cir-  
culi AC. positoq. B. centro intervallo AB. facito semicir-  
culum ADE. Tum supra C. erigo perpendicularam CD.  
quousq. tangatur circumferentia in puncto D. & connecto  
BD. Dico CD. esse quesiti circuli diametrum. Quoniam C.  
angu-





angulus rectus est quadratum subtenſæ BD. æquale est quadratis BC. CD. & quadratum BD. est æquale AB. quia ex centro, ergo quadratum CD. tanto minus est quadrato BD. quantum quadratum BC.



Quod ſi voles alio modo efficere hac ratione aſſequeris.  
Sic

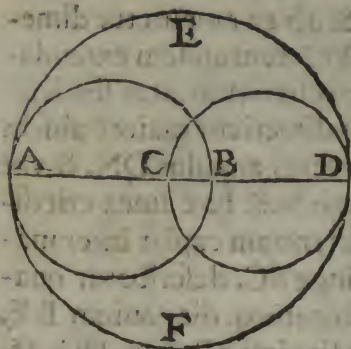
ELEM. CVRVIL. LIB. I. 17

Sit dimetiens maioris circuli CB. & ab ea amputetur dime-  
 tiens minoris circuli CD, & linea BC tantundem extenda-  
 tur ad A, & puncto C facto centro circumducatur semicir-  
 culus ANB. & à puncto D, vbi minor dimetiens maiorē abscin-  
 dit, erige super transversam AB ad rectos angulos DN. & vbi  
 DN periferiam secat ANB. istuc pone N. & hæc linea erit di-  
 metiens circuli inueniendi, qui differentiam capiat inter ma-  
 iorem, & minorem circulum. Ex linea BC. describatur qua-  
 dratum per 46. p. E. & sit CEBF. ducaturq. diagonium BE.  
 & per D punctum descendat paralellas ipsi BF. sitq. DG. se-  
 cabitq. diagonium in K & per K signum excitetur alter pa-  
 ralellus ad AB, & sit HMKL. & ex A ad H. ducatur alter  
 paralellus ipsi CM. Quoniam supplementum CK. supple-  
 mento KF. per 43. 1. est æquale, addatur commune quadra-  
 tum DL. erit CL æquale DF. sed quia AM est æquale MB  
 parallelogrammo, quia AC. & BC. sunt æquales, ergo  
 AM parallelogrammum ipsi DF parallelogrammo est æqua-  
 le, addatur commune CK. erit totum AL æquale gnomoni  
 MLF. sed quoniam MLF est excessus maioris quadrati  
 CBEF. super minorem MKEG. & quadratum lineæ DN  
 est æquale quadrangulo AK. & ex consequenti gnomoni  
 MBF quæ est differentia vtriusque quadrati; ergo DN circ-  
 ulus est differentia duorum inæqualium circulorum, quæ erat  
 demonstrandum.

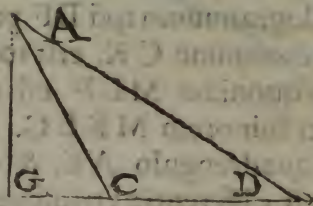
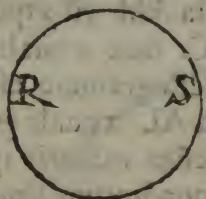
Datis tribus circulis, duos à maiori, qui  
 duobus circulis laxior sit, subduce-  
 re, & circulum dare reliquo spatio  
 æqualem. Probl. 5.

SIT amplius qualem quis conficere velit circulus ADEF.  
 sintq. pro arbitrio bini circuli AB. CD. quorum areæ  
 C totam

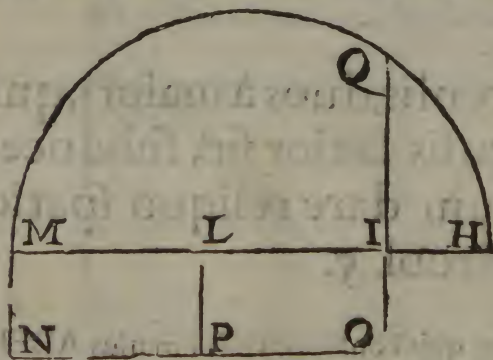




totam non contineant continentis  
amplitudinem, & ij vel in se ipsos  
flexi, vel mutuo intercisi, vt in  
exemplo, volo constringi circuli,  
qui reliquum spatium contineat,  
scilicet interceptum vacuum. Ex  
tribus AD. DC. BA. fiat triangu-  
lum ACD. quod obtusum erit  
producaturq. alterutrius maioris  
circulatus, videlicet DC. eousque  
fit productionis meta, quousque  
à trianguli supercilio, quod prædictæ lineæ incumbit, lineæ



ad perpendiculum descendat, sitq. AG. His perfectis ex-



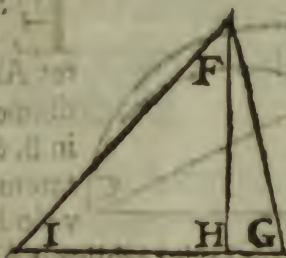
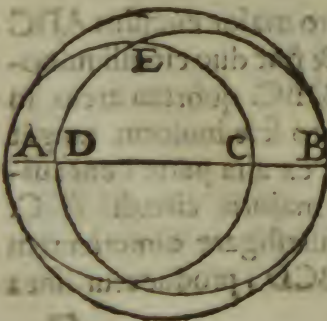
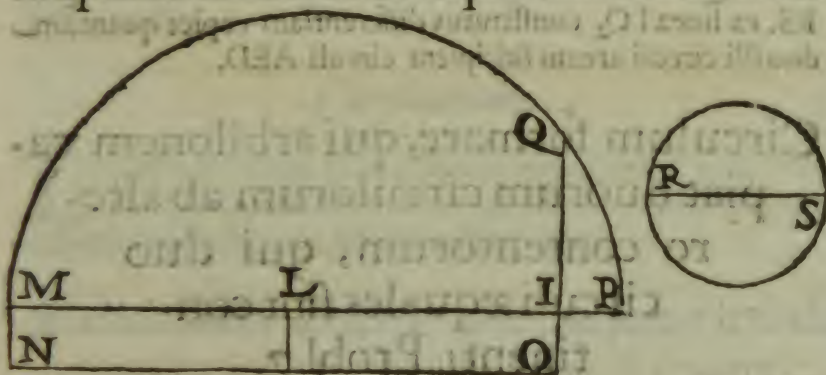
circuitiois, arcum MQH. elongeturq. OI. inferaturque  
coeun-

truatur parallelogrā-  
mum, cuius produ-  
ctus latus sit ex CD.  
geminata, & sit ML.  
LI. breuius ex CG.  
sitq. MN. IO. & pro-  
ducatur MI. donec  
æquetur IO. & sit  
IH. & extremæ lineæ  
ora terminentur per



corum lineæ cum arcu litera Q. sic ex linea IQ. fiat circulus RS. capiens iam dictam differentiam. Quoniam angulus ACD. est obtusus, quadratum lineæ AD. maioris circuli superat quadrata DC, CA, minorum circularum per rectangulum comprehensum ex DC. & CG. his per 12. 2. Euclid. & ex his constitutum rectangulum MI, & diametere QI. capiet comprehensam aream, ex qua circulus RS. quæsitam differentiam continebit.

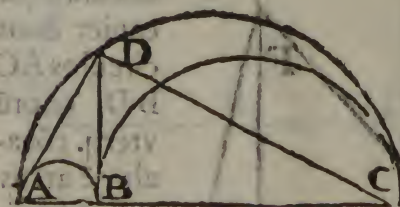
Datis tribus circulis duos à maiori, qui duobus circulis angustior sit, subducere, & circulum dare reliquo spatio deficienti æqualem. Probl. 6.



Est AEB. circulus, qui capiet duos circulos AC BD. quorū uterq. cōcauitatē arcus capiētis cōrīgat, suisq. C 2 ar-

arcis continentis areā excellent, vestigandus est circulus, qui differentiam excellentis areæ excipiat. Fiat triangulum ex tribus lineis AB. AC. DB. per 22. 1. Euclid. & sit GFI. qui erit acutus, cadat ex apice F. trianguli in substratam basem G I. orthogonaliter linea FH. & ubi eam abscindit, illic fige literam H. Porro ex geminata base GI. & linea GH in se ductis, fiat parallelogrammum MO. & superior linea MI procurrat quousque sit æqualis IO. & sit P. Mox partire intervallum MP. per æqualia in D. & ex D centro describe semicirculum, elongeturq. linea IO. quousq. attingat arcum MP. in Q. & IQ dimetiens erit futuri circuli quasitam differentiam capientis. Quoniam quadratum FI. minus est FG. GI. quadratis tantum, quantum rectangulum bis sumptum ex linea IG. GH. per 13. 2. Euclid. quod erit NI. & linea IQ. erit dimetiens continens aream NI. circulus igitur RS. ex linea IQ. constitutus differentiam capiet quantam duo illi circuli aream suscipient circuli AED.

Circulum formare, qui arbilonem capiat duorum circulorum ab altero contentorum, qui duo circuli æquales sint continenti. Probl. 7.



ESTO maior circulus ADC & sint duo circuli minores AB. BC. quorum arcus in diametro sese inuicem tangant in B. & ex alia parte concavitatem maioris circuli A. C. volo inuestigare dimetientem circuli, qui aream capiat arbilonis ABCD, producaturs linea

ex



ex mutuo circulorum contactu B. donec rotundationis maioris circuli aream tetigerit BD. dico eam esse diametrum futuri circuli, qui arbilonis ABCD. aream continet. Hanc constructionem presenti demonstratione suffulciemus. Quoniam linea AC. secta est in puncto B. quadratum, quod fit ex AC. æquale est quadratis, quæ fiunt ex AB. BC. & parallelogrammo, quod bis fit ex CB. BA. ex imperio 4. 2. Euclid. Sed parallelogrammum ex CB. BA. est æquale quadrato DB. circulus ergo ex DB. est æquale arbiloni ABCD. quod quadratum ex DB. æquale sit quadratis AB. BC. patet etiam ex 17. 6. Euclid. Vel quoniam circulus ex DC. æqualis est duobus circulis ex DB. BC. quia B. est angulus rectus, & circulus ex DA. circulis ex AB. BD. ergo circulus ex AC est æqualis duobus circulis AB. BC. & duobus circulis ex DB. qui in eo continentur, arbilon igitur ADCB. ex circulo DB. constat.

*Corollarium.*

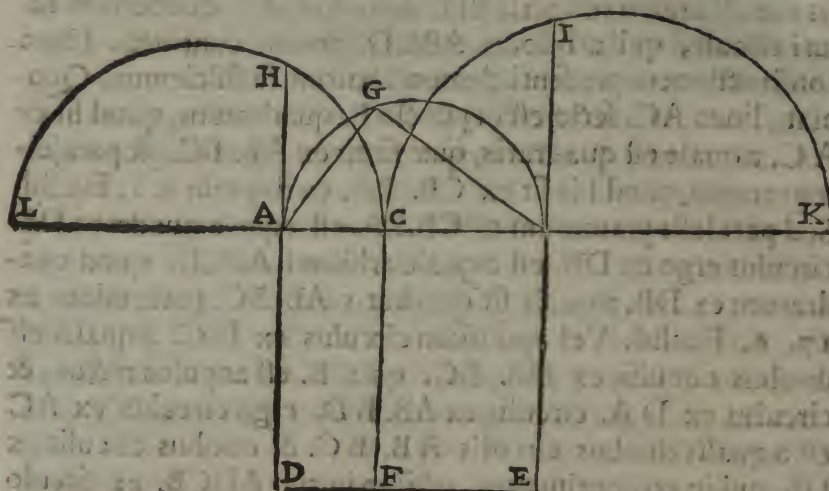
**E**X hoc provenit dato arbilone posse illico dari circulum ei æquale, scilicet lineam erigendo ad duorum semicirculorum coniunctione ad circumferentiam.

Si diameter secetur. vtcunque, circuli, qui fiunt ex tota, & singulis partibus continentur, æquales sunt ei, qui à tota fit circulo.

**Probl. 8.**

**F**iat quadratum ex linea AB. & extendatur AB vsque ad K. & sit æqualis AB. & supra CK fiat circulus CIK. & ex alia parte BA extendatur in L, & sit æqualis AB. & super





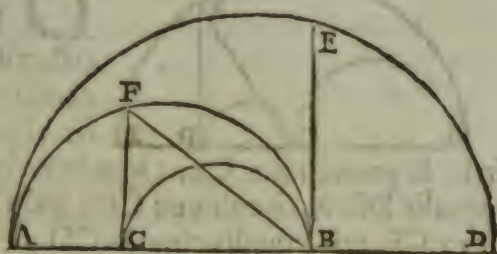
per CL. formetur circulus, & sit CHL. & supra AB fiat alter circulus AGB. & ex C extendatur parallellus ipsi AD. BE. & sit FCG. extendaturq. DA ad H. & EB ad I. ducanturq. ex G. GA. GB. Quoniam quadrangulum ex KB. BC. est æquale quadrato BI. & quadrangulum ex LA. AC. est æquale quadrato AH. coheant ipsæ BI. AH. in circumferentia AGB. in puncto G. quia in circumferentia ad rectum angulum: ergo quadratum ex AB. duobus quadratis AG. GB. æquale erit, & sic de circulis, vel aliter.

Quoniam per præcedentem diametrum diuisa bifariam in C. quadratum ex AC. & CB. & rectangulum bis contentum ex BA. AC. est æquale quadrato AB. sed rectangulum bis contentum ex BA. AC. est æquale quadrato ex CG. sed quadratum ex BC. CG. & quadratum ex AC. CG. sunt æqualia quadratis ex AG. GB. & quadratum ex AG. GB. sunt æqualia quadrato ex AB. ergo ostendimus, quod intendebamus, & est secunda secundi Euclid.

Si

Si diameter secetur vtrunq; circulus ex tota, & eius parte contentus æqualis erit circulo, qui ex partibus continetur, & eius quod ex prædicta parte fit circulus. Propos. 9.

**S**it diameter AB. secta vtrunq; in puncto C. dico circulum ex AB. BC. contentum æqualem esse circulo ex BC. CA. contento, & circulo CB.



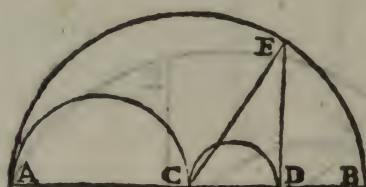
Extendatur AB. in D. & fit BD. æqualis ipsi BC. & super ACBD. fiat circulus, & fit AED, & ex puncto B. eleuetur perpendicularis vsque ad E. Idem fiat ex altera parte. Supra AC. & CB. duo circuli, & ascendat ex C perpendicularis CF vsque ad semicirculum AFG, extendaturq; FB.

Quoniam quadrangulum, quod fit ex AB. BD. æquale est quadrato, quod fit ex BE. & quadrangulum, quod fit ex BA. AC. æquale quadrato ex CF. sed quadratum ex FG. æquale est quadratis FC, CB. quia C angulus est rectus ergo circulus ex AB. BC. quod est BF æquale est circulis ex CB. & qui fit ex BC. CA. & est 3. 2. Euclid.

Si



Si diameter secta fuerit in partes æquales, & inæquales circulus ex inæqualibus partibus contentus vna cum eo, qui fit ex linea, quæ inter sectiones interijcitur æqualis est circulo, qui fit à dimidia. Prop. 10.



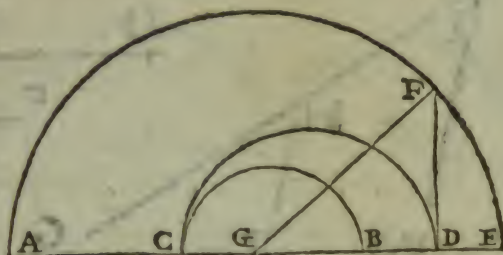
**D** Escribatur circulus ex Diametro CA. alter ex AB. alter vero ex CD. ex D. puncto erigatur perpendicularis vsque ad circumferentiam in E. & protrahatur CE. Quoniam circulus ex AD. DB. est æqualis DE. & circulus ex CD. ipsi CD. ergo circulus, qui fit ex CE. erit æqualis circulis CD. DE. sed CE. est æqualis CA. quia ex centro ad circumferentiam, ergo circulus ex duabus inæqualibus partibus compositus AD. DB. qui est DE. & circulus CD. vtrique æqualis est circulo ex dimidia CA. compositus, & est 5. 2. Euclid.

Si diameter bifariam secetur, eiq. in rectum adijciatur quædam recta linea, circulus ex tota diametro cum adiecta tanquam ex vno diametro, vna cum circulo dimidiæ æquales sunt circulo ex dimidia, & adiecta tanquam ex vna diametro descripto. Prop. 11.

**S** It diameter AB. secetur bifariam in C. & ei in longum adijciatur linea BD. dico circulus descriptus ex AD. DB. vna

DB. vna cum circulo  
C B. æquales esse cir-  
culo, qui fit ex CD.

Lineæ AD. adijcia-  
tur DE, quæ sit æqua-  
lis DB, & centro G in-  
teruallo GE describa-  
tur circulus AFE. &



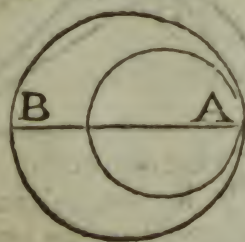
ex puncto D linea ad rectum erigatur vsque donec circuli  
circumferentiam contingat, & sit DF. & erit quadratum  
quadranguli AD. DE. & puncto D, linea DC secetur CB  
æqualis, & erit GD. & connectantur puncta GF. linea GD  
est æqualis CB. ex constructione. Quoniam linea AC. est  
æqualis lineæ CB. & CB ipsi GD. adijciatur ipsi AC. com-  
munis CG. & linea DE. est æqualis BD. ex constructione,  
ergo CD. ipsi GE. & angulus ad D. rectus est, valet ergo  
quadratum GF. quadrata GD. DF. ergo quadratum GF. va-  
let quadratum CD. quod demonstrandum proposueramus,  
& est 6. 2. Euclid.

A dato circulo alium in datam pro-  
portionem abscindere.

Prop. 12.



STO datus circulus AB.  
volo alterum construc-  
re, vt ad eum datam  
proportionem habeat,  
sitq. data proportio CD.  
ad EF. scilicet sesquialtera. Iungantur  
angulo binæ lineæ, quarum vna GH.  
sit æqualis lineæ CD. protendanturq.  
quousque HI sit æqualis EF. Mox alteri lineæ æquetur dia-

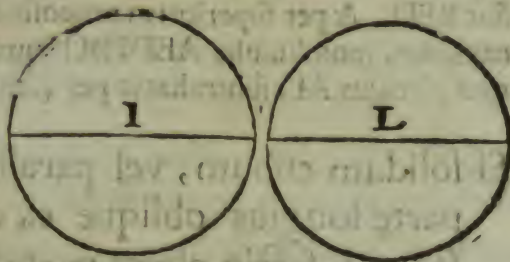


D meter



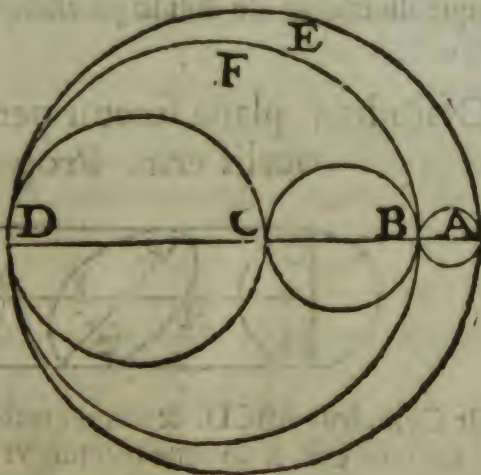


prodac o GE.EF.dico  
duos circulos duarum  
dimetientium GE, EF.  
esse æquales duobus  
dimetientibus G.H,  
HF. & proinde circu-  
lis I, L. Quoniam an-  
gulus H est rectus,  
quia ad circumferen-  
tiam, ergo quadrata G.H. HF. sunt æqualia quadrato GF.  
& quadratis GE, EF. etiam æqualia quadrato GF. & quæ  
æqualia vni tertio æqualia inter se, ergo circuli I, L. sunt  
æquales AB. CD.



Circulum formare, qui capiat arbilonem trium  
minorum circulorum, ab imo maiori conten-  
torum, qui tres circuli æquales sint diametro  
continentis. Prop. 14.

EST O circulus  
AED. cuius  
dimetiens AD. tri-  
bus circuli diame-  
tris intercidatur DC  
CB, BA. postulamus  
circulum formare,  
qui arbilonem, vel  
interceptam arcum  
à maioris circuli cō-  
cauitate, & mino-  
rum conuexitate  
conineat. Ex BD  
Diametro circulus



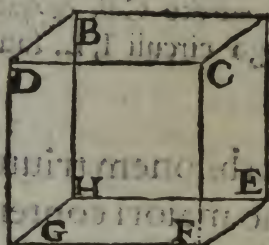
D 3 fiat



fiat BFD. & per superiorem propositionem 7. arbilon BFDC capiatur, mox lunulæ AEDFBC quantitas cognoscatur, à qua circulus AB. subtrahatur per 4. nostram, & sic de cæteris.

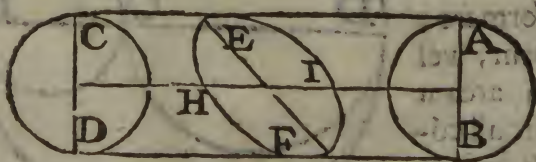
Si solidum cubum, vel parallelipedum altera parte longius oblique ex oppositis lateribus secetur sectio altera parte longius erit.

Propos. 15.



**E** Sto solidus cubus ABCDEFGH & secetur à plano BDEF. oblique ex oppositis cubi lateribus BD. EF. dico BDEF. esse altera parte longius. Quia DG, GF, æqualis est DF autem subiaccens linea est æqualis duobus quadratis DG. GF. ergo longior BD. quæ ipsi DG æqualis est, idem dicendum de altera parte BH. HE. quia BE, maior est BH. HE. Igitur BDEF. altera parte longior est. Idem quoque dicendum de solido parallelipedo altera parte longiori.

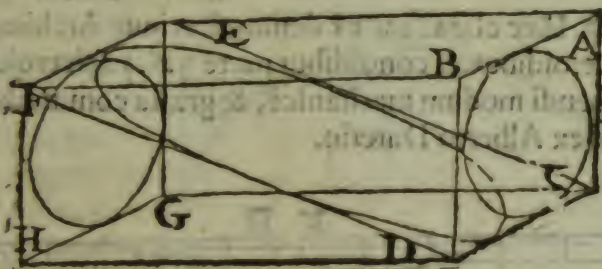
Si Cylindrus plana secetur per obliquum sectio oualis erit. Prop. 16.



**S** It Cylindrus ABCD. & secetur rectè ABG. sectio AGB. circulus erit, si oblique secetur, vt in IEHF. sectio sphærois erit ex ea quæ Serenus probauit in suis Cylindricis.

Si

Si intra solidum paralleli pedum altera parte longius cylindrus inscribatut tangens sui circuli basis latera eius quadrati, & paralleipedum solidum obliquè fecetur ea proportio erit circuli quadrato, quam sphærois figura ad suum altera parte longius. Prop. 17.

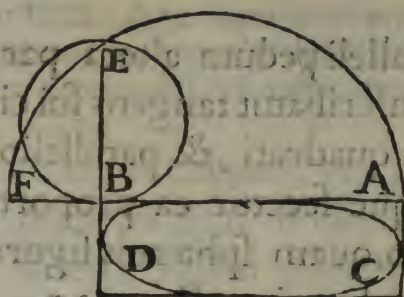


**S**it paralleipedū solidū altera parte longius ABCDEFGH & sint cylindri in eo descripti bases ABCD. EFGH. circuli in ea descripti ABCD. EFGH. & planum obliquè secans illud sit CDEF. & sphærois in eo descripta CDEF. dico sphæroidem intra se descriptam eandem habere proportionem ad suam figuram altera parte longiorem, quam circulus ABCD. ad suum quadratum ABCD. cuius demonstrationem omittimus: nam ex his, quæ Euclides in suorum elementorum, 12. & Archimedes in 31. præpositione descripserunt, demonstratur.

Data sphæroide circulum eiusdem areæ describere. Prop. 18.

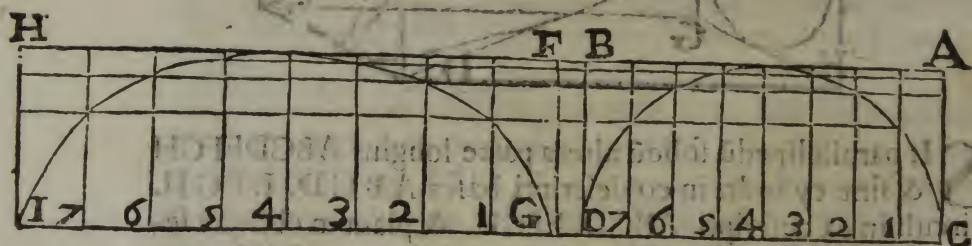
**E**Sto data sphærois ABCD. in eo circulum eiusdem spatij. Circa datam sphæroidem quadrangulū circumscribatur





batur ABCD. & latus AD  
prolongetur vsque ad F. vt  
BF. sit æqualis BD. Et cir-  
ca AF. semicirculus descri-  
batur, elongeturq. BD. do-  
nec circumferentiam fe-  
riat, & sit in puncto E. di-  
co circulum conscriptum  
circa BE. diametrum con-  
tinere aream sphæroidis

ABCD. Hæc clara sunt ex demonstratione Archimedis libro  
de sphæroidibus, & conoidibus parte 5. 6. 7. sphæroidem idem  
describendi modum mechanicè, & gratia commoditatis pro-  
ponam ex Alberto Durerio.



Describe quadrangulum in duplo triplo, aut sesquialtero,  
& sit in circulo supra AB. infernè CD. cuius latus CD. diui-  
de in puncto E. per medium, ac posito vno circini pede in pun-  
cto E. intervallo EC. ducatur per superiorem partem vsque  
ad D. contingeret hic arcus lineam AB. deinde partire lineam  
CD. in octo æquales partes, & ex singulis diuisionibus pro-  
trahe sursum parallelas in nuper descriptum arcum. Deinde  
fac iuxta quadrangulum ABCD. adhuc alium quadrangu-  
lum æqualis altitudinis, sed longirudinis quantæ volueris cu-  
ius superior linea FH. inferna vero GI. & seca id quoque in  
octo partes æquales, vt prius, postea producito ex singulis se-  
ctionibus sursum lineas parallelas, deinde ex singulis inter  
sectio-

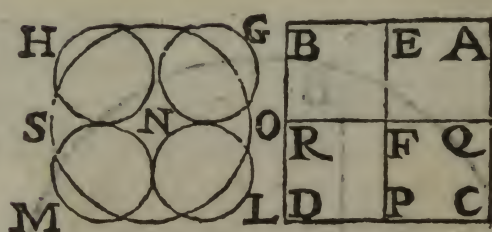


tunda



cunda (nam CF sumpta est æqualis EB.) ad parallelogram-  
mum E O. supra tertiam E C. quod simile; similiterq. de-  
scriptum.

Si circuli diameter bifariam secetur, & ex vna  
parte circulus fiat hit erit totius pars  
quarta. Prop. 20.

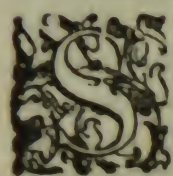


**R**ationes circu-  
lorum sequun-  
tur rationes quadra-  
torum eis circum-  
scriptorum, vel in-  
scriptorum, & quæ-  
admodum, si qua-  
drati diameter diui-

datur quadratum ex vna parte, erit quarta parte totius, ita, &  
circulus; Exemplum latus AB. quadrati AD. diuidatur bi-  
fariam in E. dico quadratum ex AE. quod est AF. est AD  
quadrati pars quarta. Trahatur EP parallela, ipsi AC. &  
QR. ipsi AB, & erunt quatuor parallelogramma rectangula,  
& si aliter probari posset rationem recitabo apud Platonem  
in Memnone. Socrates enim puerum hoc modo docet. Sic  
bipedalis linea AB. dico suum quadratum esse quatuor pe-  
dum AQ erit vnus pedis, erunt dico quadrata QF. FR. sit &  
altera pars CD duos pedes longa vnum alta C. erunt enim  
duo quadrata CF. FD. tota igitur quatuor erit pedum. Sit  
ergo circulus OILM. cuius diameter ON S. diuidatur bifa-  
riam in N. ex quantitate ON. quatuor circuli inscribantur,  
dico quatuor hos circulos toti æquales esse. Ratio ex supe-  
riori pendet: nam & circuli se habent ad quadrata, vt eorum  
diametri.

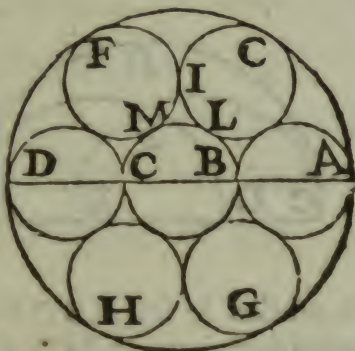
Cir-

Circulorum vacua metiri, quando maior minores contineat. Prop. 21.



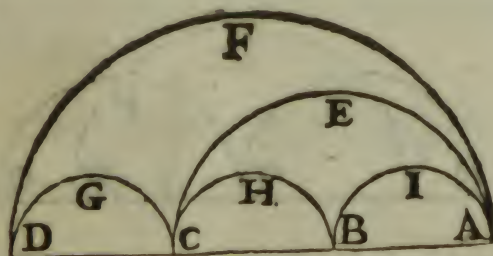
IT magnus circulus AEFDHG, cuius diameter AD, diuidatur in tres partes, & in eo fiat tres circuli AB, BC, CD, & supra duo alij, & duo infra inscribantur; nam sex circuli æquales intra vnum inscribuntur ex 15.

4. Euclid. & ex præcedenti totus circulus nouem circulos continebit: nam diameter trifariam diuisa est, sunt intus septem contenti, ergo omnia vacua duo erunt circuli cuius 3. pars erit scalprum EIF. cum suo residuo ILM.



Arbilones per circulares figuras metiri. Prop. 22.

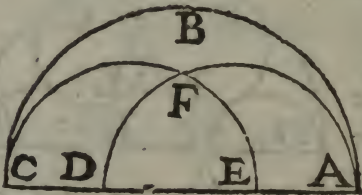
Si arbilon primū AFDGCHBIA. inuestigādum quot circulos capiet, qualis AB. Ex præcedēti semicirculus AFD nouem capiet semicirculos qualis AIB si substuleris AIB, BHC, CGD, erit arbilon reliquum sex



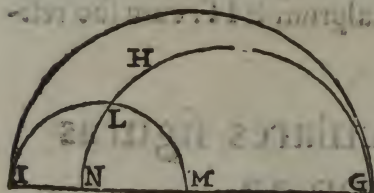
E semi-



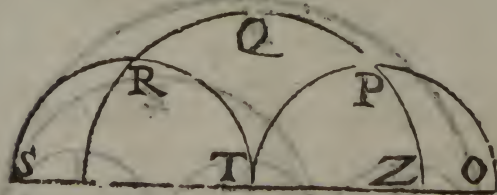
semicircularum. Si quærimus arbilonem AFCHBI. erit semicirculus AEC quatuor semicircularum qualis AIB, demptis duobus AIB, BHC, erit arbilon duorum semicircularum. Si quærimus arbilonem AFDGCEA, erit ex iam dictis quatuor semicircularum.



At si semicirculus maior ABC capiens duos semicirculos AED, EFC, ut docuimus in prima nostri, dico arbilonem ABCFA esse æqualem duplato EFD, quod ex figura patet: nam quod replicatur in figura EFD deficit arbilon in sua ABCF. Vel quarta pars dupli BEA. est æqualis semicirculo AFD. pars externa EFD est æqualis interiori corniculari angulo FBA.



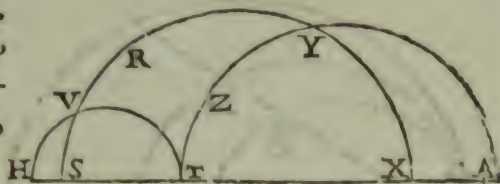
Idem eveniet in figura GHI: nam duo semicirculi GHLN, & MLI. per secundam nostri capiunt aream continentis circuli GHI. Vnde duplatum MLN. est æquale arbiloni GHLIHG.



Potest etiam evenire, ut arbilon medium PQRT est æquale duobus extrinsecis circuli partibus OPZR S. ex superiori ratione.

Idem

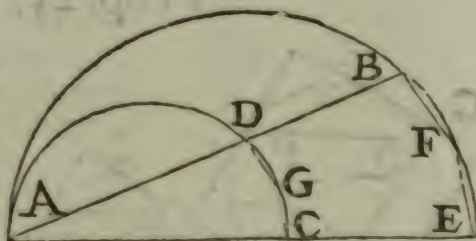
Idem eueniet in hac  
postrema, vt arbilon  
YZTVRY. fit æquale  
duobus circuli extrinse-  
cis partibus A Y X,  
SVH.



Siduo vel quamplures circuli in fine  
diametri se tangunt à contactus au-  
tem puncto ducatur linea eos secans  
arcus secti inter se similes erunt.

Prop. 23.

**S**Int duo circuli  
ABE, ADC se  
mutuo tangentes in  
fine diametri A, &  
ducatur recta linea  
ADB, secans arcus  
ADC, in D, & ABE,  
in B, qui quidem ar-  
cus bifariam secen-  
tur, quia anguli in circulo oppositi per 22. 3. duo æquales re-  
ctis duobus in 2. ergo angulus BGC, & BFE æquales sunt  
cum eodem BAC angulo iuncto.



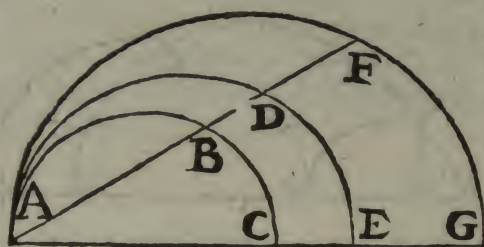
Data circuli portione eam multipli-  
care, Prop. 24.

**S**It data circuli portio AB, quam volo duplare & fit,  
eius circulus ABC, & fit semicirculus ADE du-  
plus

E 2

plus

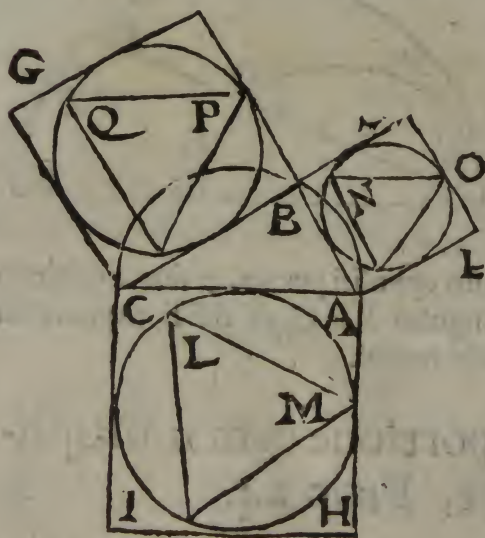




plus dati. Per primam nostri, linea A B trahatur longius in D, & si voverimus quadruplare sit circulus AFG quadruplus & linea AD in F extēdatur, dico portionem DA ipsius BA duplam, & FA ipsius BA quadruplam cuius ratio pendet ex anteriori.

Ex duabus portionibus similibus vnā similem facere, vel subtrahere.

Prop. 25.



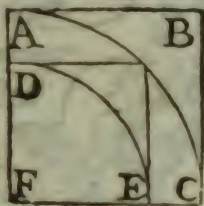
Int duę inæquales circuli portiones PQ. ON. sed similes, & sit vnaquęque tertia circuli pars per 25. 3. Eucl. & sint P Q G, O D N; circa quos describantur quadrata B G, E B, vel eorum diametri, & iungantur ad rectum angulum ABC, & secundum AC describatur quadratum, & in eo circulus MLI, & sit ML latus æquilateri trianguli. Portio ML erit æqualis iam dictis dua-

duabus portionibus per ea quæ in 3. Euclid. probantur. Vel si ex ML voluerimus portionem PQ subtrahere, è recto quadrato AC, ac supra AC semicirculo descripto, ponatur latus quadrati BC, & eius latus BA latus quadrati portionem similem continentis. Et sic possumus ex pluribus portionibus vnam facere, & omnia illa, quæ de integro circulo retulimus.

Datum semicurvilineum triangulum  
duplare, subducere, vel è duobus  
similibus vnum facere.

Prop. 26.

**S**it semicurvilineum triangulum DGE, quod volo duplare, & sit circuli quarta pars FDE, fiat etiam circuli dupli pars, & sit AGC, circa eam quartam etiam quadrati partem circumscribo ABCF, dico triangulum semicurvilineum ABCG. duplum esse DGE, Quia quadratum ABCF duplum est DGFE inscripta portio proportionalis erit. Et sic subtrahere, & ex multis vnam facere poterimus ex supradictis.



Eodem modo triangulum DEG duplare poterimus, quod est æquale iam dicto: nam quadrati dimidium BHA est æquale BAG, si dematur portio BIA, æqualis BCG. remanet triangulum BAG. æquale BHA, iam



dicto.



dicto. Vnde si voluerimus prædictum EDF semicirculi-  
 neum triangulum duplare, duplato quadrante  
 HABG, protractoq. diametro BA.  
 circulus duplus BIA, qui  
 erit BC descri-  
 batur  
 BC, & erit triangulum ABC da-  
 plum trianguli  
 EDF.



39

IO. BAPT. PORTÆ  
NEAPOLITANI  
ELEMENTORVM CURVILINEORVM  
Liber Secundus.

A X I O M A T A.

I.

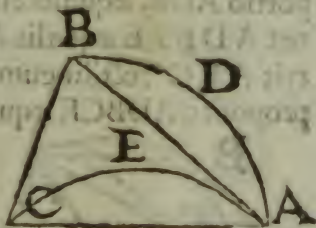
**S**i eidem addideris, quod prius dempseris, quantitas æqualis erit.

II.

Si nota quantitas à nota subtrahatur, quæ remanet nota erit.

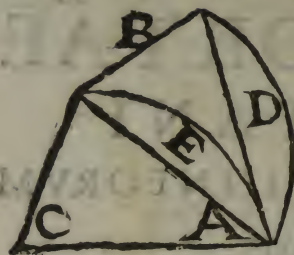
Triangulum semicurvilineum ex æqualibus, iisdemq. circumferentijs compositum quadrare. Prop. I.

**E**sto triangulum quodpiam semicurvilineum  $ADBC E$ , Aequalibus nimirum iisdemq. circumferentijs  $ADB$ ,  $AEC$ , & rectæ  $BC$  basi constituta volo illud quadrare. Ducatur linea  $AB$ , &  $AC$ , aio aream trianguli semicurvilinei  $ADBC E$  esse æqualem triangulo rectilineo  $ABC$ . Quoniam circumferentia  $ADB$  est æqualis portioni  $AEC$ , ablata  $ADB$ , repositaq. in  $AEC$  æquale remanet triangulum rectilineum  $ABC$  semicurvilineo per primum axioma nostrum.



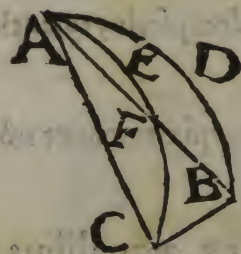
Vel



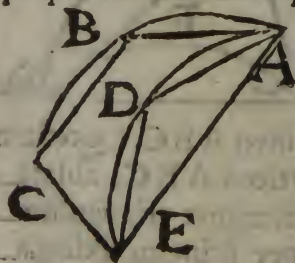


Vel fiat triangulum æquale rectilineum  $ABC$ , & sit  $AFC$  ex 22.1. Euclid. erit semicurvilineum triangulum  $ADBCE$ , æquale triangulo semicurvilineo  $AECF$  dempta communi portione  $AEC$  remanet rectilineum  $AFC$  triangulo semicurvilineo æquale  $ADBCE$ .

*Alter Casus.*



**A**T si triangulum  $ADBCE$  angustius erit, & portiones lineæ neutiquam intactas circumferentias relinquent, sed per medium transibunt, eadem operatione idem assequi poterimus. Sed quo res dilucidior euadat, rem exemplo complectemur. Esto triangulum  $ADBCE$ , & circumferentia  $ADB$  æqualis sit  $AFC$ , trahanturq. rectæ lineæ  $AB$ ,  $AC$ , & secet  $AB$  basis  $ADB$  circumferentiam  $AEC$ , aio rectilineum  $ABC$  æqualem semicurvilineo  $ADBCE$ . Quoniam portio  $ADB$ , æqualis est  $AEC$  dempta communi  $AEF$ , remanet  $ADBF$  æqualis  $AFC$ , apponatur vtrique areola  $FBC$ , erit  $ABC$  rectilineum triangulum semicurvilineo triangulo proposito  $ADBCE$  æquale.



Vel ad eadem præstanda possumus easdem circumferentias in plures partes diuidere, nempe binas, ternas, quaternas, vt  $ABC$  circumferentiam in  $AB$ ,  $BC$ , &  $ADE$  in  $AD$ ,  $DE$ . Vnde exclusæ partes  $AB$ ,  $BC$ , inclusis  $AD$ ,  $DE$  erit area rectilinea  $ABCEDA$  æqualis semicurvilineo  $ABCEDA$ .

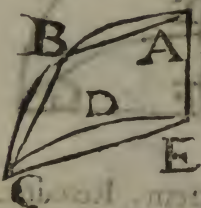
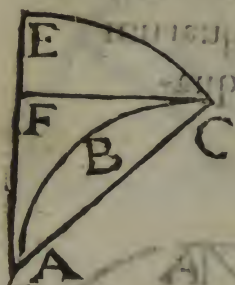
Trian-







demptione abiit, tota ex repositione substituta est.



Vel potest transpositis lineis alio modo triangulum semicurvilineum constitui sit circumferentia dupli CDE retro CBA ante, tunc ex puncto C. super basim AE cadat perpendicularis CF, & connectatur CA, & sic triangulum semicurvilineum ABCDE rectilineo FCA parem iri. Ratio in superiori.

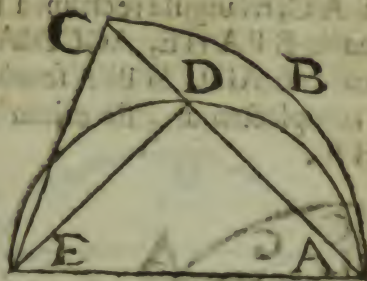
At si vt diximus ex varijs, & inæqualibus circumferentijs orbiculata triangula composita erunt, tunc mente concipiendum, si circulus duplus alteri sit, subdupli duæ circumferentiæ partes, vni dupli respondent, si quadrupli quatuor, & sic deinceps. Esto verbi gratia circuli dupli circumferentia EDC, & sit octaua suæ circumferentiæ pars respondet duobus octauis subdupli circuli ABC. Diuidatur ambiens linea ABC bifariam in B, & trahatur AB, BC, EC, & erunt duæ AB, BC portiones, vni EC æquales, & sic vna EDC, duas illas AB, BC absumet. Vnde si triangulum semicurvilineum duabus octauis circumferentiæ partibus decrescimus, AB, BC augemus vna EDC, & sic par pari referemus.

### *Alter Casus.*

**P**otest & aliter euenire; sit triangulum semicurvilineum ABCED, & sit ABC quarta dupli circuli, & ADE semicirculus subdupli, docebimus quomodo possis rectilineum triangulum æquale semicurvilineo facere. Trahatur ex puncto per medium circuli ADE vsque ad C, & sit linea ADC, & linea DE. Erit triangulum semicurvilineum ABCED æquale

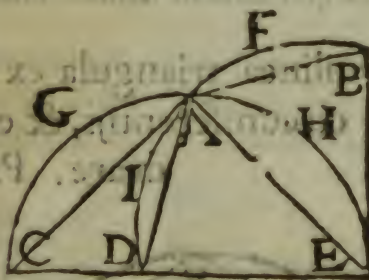


equale rectilineo DCE. Quoniam portio ABC est dupla ipsius AD per 19. primi nostri, & huic nempe portioni AD æqualis DE. dematur dimidia portio ABCD, addatur DE compar, remaneatq. communis areola DCEF, utrique sic enim rectilineum triangulum DCE æquale semicurvilineo ABCED, & sic excessus unius alterius defectu rependeretur. Sic & in alijs notis circumferentijs quadruplis quintuplis eodem Methodo uti poteris.



Semicurvilinea triangula ad verticem constituta ex eisdem, & æqualibus circumferentijs, vel ex æqualibus nota quadrare. Prop. 3.

SI duo semicirculosa triangula ad verticem constituta ex eisdem, & æqualibus circumferentijs fuerint ductis à vertice ad bases rectis lineis, erunt rectangula circulosis equalia. Si primam huius libri leges non secus esse inuenies, quā diximus. Si acciderit, ut circumferentiæ eadem ad verticem sint inæquales, sed in id conueniant oportet, ut dextra interior sinistro exteriori æqualis sit. Sint inæqualia triangula se inticem decussantia BAE, ACD, segmenta sint æqualia, ut BFA, AID, & EHA, AGC, tunc protractis rectis BA, AD, F 2 AE,



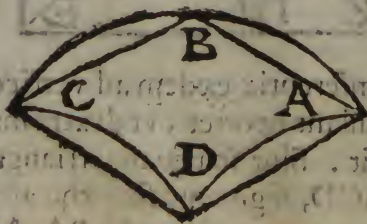


AE, AC, triangula rectilinea BAE, ADC, erunt circuloſis æqualia BFAHE, AGCDIA. Quoniam ſegmentum BFA æquale eſt AID. Si BFA ſeorſum expellimus, & AID ſua vice complectemur, ſic etiam reiſcimus AGC reponimus EHA.



At ſi fuerint duo ſemicuruiſine triangula BEAFD, & AGCIH conſtituta ad verticem A ex inæqualibus circūferentijs notis quarum DAC ſit circulus duplus ipſius BAI. Trahantur duæ lineæ perpendiculares ex A ad CI. & ſit AL, & AM ad BI. & binæ aliæ rectæ BA, AI, dico rectilinea triangula ALI, ABM, ſimul iuncta æqualia eſſe. Semicuruiſine BEAFD, CGAHI. Quoniam periferia DAC eſt circuli dupli quarta, & BAI ſubduplus ſemicirculus, duæ ſemiportiones AFDM, AGCL, abſument duas portiones BEA, AHI demptis igitur BEA, AGCL, reſiſtiſq. AHI. AEBM, rectilinea triangula BAM, LAI, æquiualebunt ſemicuruiſine iam dictis.

Curuiſine triangula ex eiſdem & æqualibus circumferentijs, & ex varijs notis quadrare. Prop. 4.

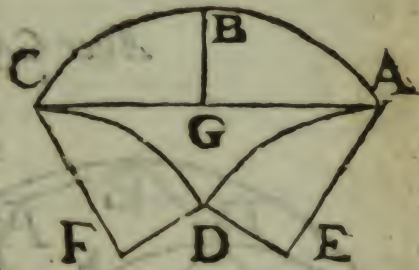


Sto curuiſineum triangulum ex tribus circumferentijs ABC, CD, DA, eiſdem curuiſine, ſed ABC, dupla AD, DC conſtitutum quod quadrare intendimus. Dico protra-

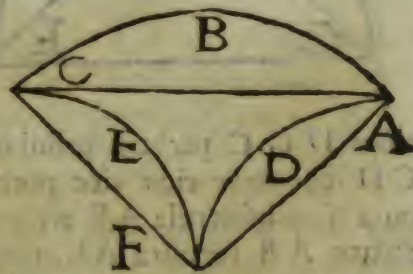


etis æqualibus subtenfis AB, BC, CD, DA quadrilaterum, rectilineum ABCD, esse æquale curvilineo ABCD. Quoniam demendo portiones AB, BC, addendoq. AD, DC, quæ simul æqualia sunt voti compos fies, vel aliud dicimus.

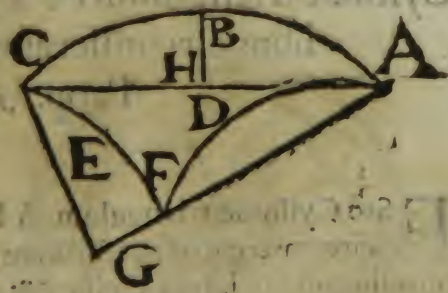
Poterimus alio modo id assequi. Protrahatur linea AC, & binas AE, ED, & lineas CF, FD, vt semiportio AED sit æqualis ABG, & DCF, & ipsi BGC, nam, duæ portiones dimidiatæ ADE, CDF æquivalent vni integræ ABC. Vna hac dempta, his additis quod diximus eueniet.



Eodem modo curvilinea triangula ex inæqualibus circumferentijs, sed altera alterius exempli causa sit dupla. Sit curvilineum triangulum ABCEFD ex inæqualibus circumferentijs, sed ABC dupla sit ADF, & FEC subtenfis lineis AC, AF, FC erit quadratum nempe binæ portiones ADF, EFC æquipollent simplici ABC, vnde illa dempta, his additis triangulum rectilineum FAC æquipollet curvilineo proposito.



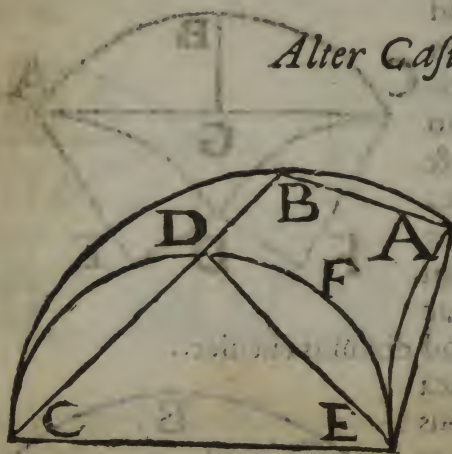
Potest contingere, vt triangulum constitutur ex varijs circumferentijs, & inæqualibus, vt FEC sit dimidia ipsius ABC, & ipsa ABC dupla ipsius ADF, sic facta semipor-





tionē CFG, æquali BHC, & subtensis AF portio ADF erit æqualis ABH. Vnde hac dempta, illis subditis triangulum rectilineum ACG erit æquale curuilineo ABCEFD.

*Alter Casus.*



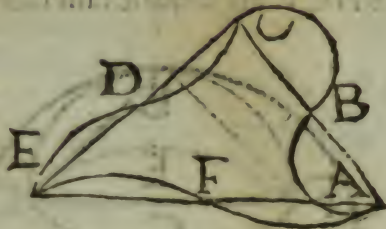
**E** Sto curuilineū triangulum ABCDFE propositum quadrādum, & circumferentiā circuli ABC sit dupla EFD GC, diuidatur circumferentiā EDC bifariam in D, & trahatur CDB erit ceratoide triangulum BHCGD æquale portioni DGC per 19. primi nostri. Vnde dempto BHCGD reponatur eius vice portio EFD æqualis DGC, & quia circumferentiā AE est æqualis, & eadem ipsius AB, ablato AB reposita AE, trapezium rectilineum ABDE erit æquale proposito curuilineo triangulo ABHCGDEF.

**Cyffoide triangulum ex æqualibus, & inæqualibus circumferentijs quadrare.**

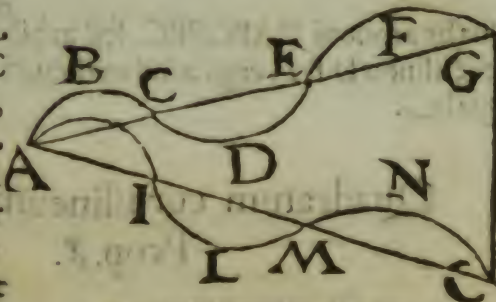
**Prop. 5.**

**E** Sto Cyffoide triangulum AFC ex tribus inæqualibus circumferentijs constitutum ABC, CDE, EFA curuilineum, & latera diuisa, & æqualibus circumferentijs con-

constituta, ut  $AB$ , sit  
 æqualis  $BC$ , &  $CD$ ,  
 ipsi  $DE$ , &  $EF$ , ipsi  
 $FA$ , unde tractis lineis  
 rectis  $AC$ ,  $CE$ ,  $EA$ , &  
 demptis tribus circum  
 ferentijs  $BC$ ,  $DE$ ,  $FA$ ,  
 & alijs tribus repositis  
 $AB$ ,  $CD$ ,  $EF$ , rectili-  
 neum triangulum  $ACE$  æquale est cyssoidi  $ABCDEF$ .

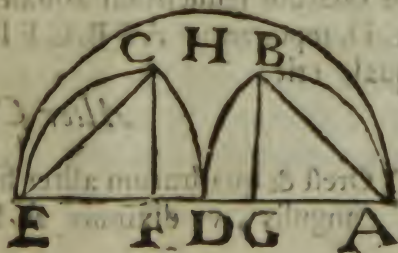


Sit quoque semi-  
 cyssoide triangulum  
 quadrangulum  $ABCD$   
 $DEFG$   $I$   $LMNO$ ,  
 ex varijs circularum  
 circumferentijs, sed  
 tamen binis semper  
 oppositis æqualibus  
 constitutum, videlicet  
 $GFE$  maioris circuli  
 circumferentia, quam  $EDC$ , &  $EDC$  maior  $CBA$ , sed  
 tamen  $GFE$  æqualis  $ONM$ , &  $EDC$ ,  $ILM$ , &  $CBA$ ,  
 $AHI$ , si à puncto  $A$  ad basim  $GO$  lineæ rectæ trahantur,  
 totum assequeris, ratio pendet ex superiori.



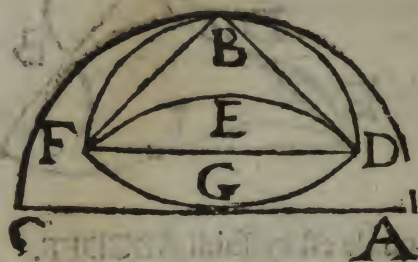
Arbilonem quadrare. Prop. 6.

Esto arbilon  $AH$   
 $ECDB$  qua-  
 drangulum, quia portio  
 $AH$  est dupla  $AB$ , &  
 $AB$  est æqualis semi-  
 portio  $BGD$ , ergo  
 ablata  $ABH$ , & re-  
 posita





posita BGD, & ablata HCE reposita DCF, rectilineum GBHCF est æquale iam dicto arbilioni.

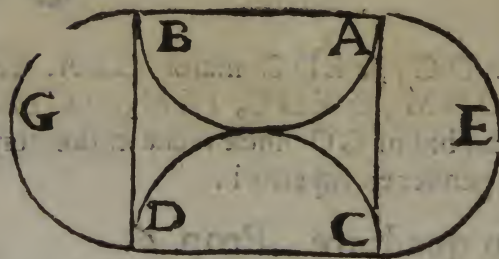


æqualis arbilioni DABGFBC, sed arbilion est æquale triangulo rectilineo DEF, ergo arbilion dictum triangulo DBF, est æquale.

Potest & alio modo probari. semicirculus ABC est duplus semicirculi DBF, ergo vacuum ABDGFBC. est æquale semicirculo, dematur ex utroque portio DEF, DGF, ergo lunula DBFE est

### Quadratum curvilineum quadrare.

Prop. 8.



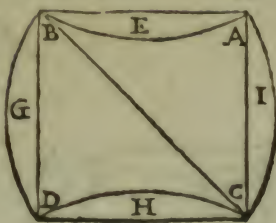
sunt quatuor semicirculi æquales inuicem, tollantur AEC, BGD, reponantur AFB, CFD, sic rectilineum curvilineo æquale erit.

### Alter Casus.

Potest & quadratum aliter fieri, ex quatuor etiam rectis angulis, ut diximus ABC. Quoniam portiones æqua-

F Sto quadratum curvilineum AFBGDFCE. trahantur quatuor lineæ ex angulis AB, BD, DC, CA, dico quadratum rectilineum ABCD curvilineo iam dicto præstabit. Quoniam

æquales sunt, & ex æqualibus circulis, ablati portionibus AIC, BGD, repositisq. AEB, CHD rectilineum quadratum ABDC, curvilineo AEBGDHCA æquipollebit.

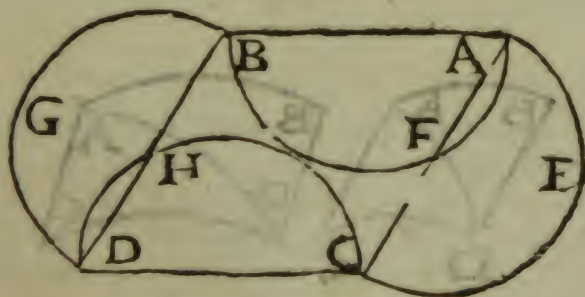


*Corollarium.*

**H** Inc patere potest quadratum curvilineum ex aduersis, & conuersis circumferentijs constitutum recta diameter bifariam secat, latus AB, lateri AC æquale est, & basis BC communis utrique, ergo triangulum CAB triangulo BDC æquale erit: igitur bifariam secat, & utrumq. ex conuexo, & concauo æquali latere constat.

Rhombum curvilineum quadrare.

Prop. 8.

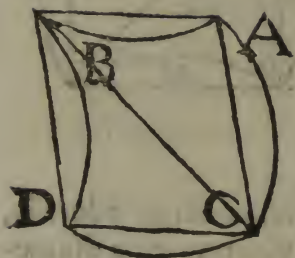


**E**T Rhombus curvilineus AFBGDHCE quadrabitur, ductis ex angulis rectis lineis AB, BD, DC, CA, nam demptis semicirculis AEC, DBG, repositisq. AFB, CHD, demptisq. portionibus HDAF, rectilineum Rhombum curvilineo æquabitur.

G

*Alter*



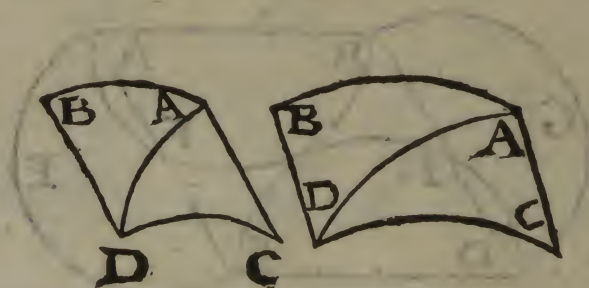
*Alter Casus.*

**P**otest esse Rhombus alio modo ex æqualibus circumferentijs AB, BD, DC, CA, & quoniam portiones æquales sunt, duabus demptis AC, CD, totidem repositis AB, BD erit rectilineo æqualis.

*Corollarium.*

**E**iusmodi etiam Rhombos recta dimetiens æqualiter secabit; nam hinc inde duo æqualia triangula constituent.

Rhombos, seu Rhomboides semicurvilineos quadrare. Prop. 9.

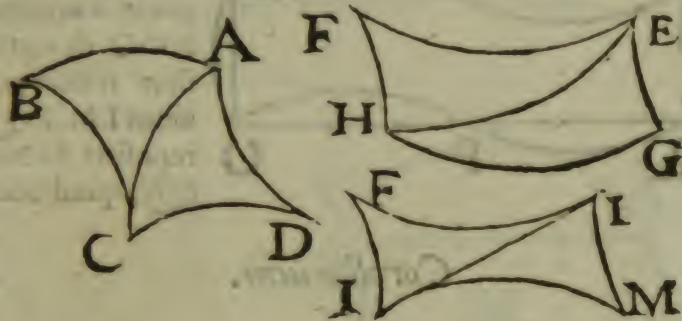


**S**emicurvilineus Rhombus, & Rhomboides facilius quadrabitur: nam portione vna dempta, altera reposita, æquales erunt curvilinei rectilineis.

*Corol-*

*Corollarium.*

**S**ed in istis, qui ex isoscelibus triangulis semicurvilineis constituuntur curua diameter circumferentiæ æqualis, & eos bifariam secabit; nam in duo æqualia isoscelia trian- gula diuiduntur semicurvilinea, vt ABD, ADC.



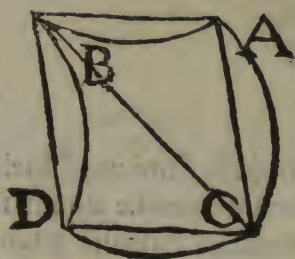
Possunt & alio modo Rhombos, & Rhomboides in Isosce- libus triangulis constitutos ex tribus conuersis, & vna auer- sa diametro per mediam diuidere, vt in Rhombo ABCD. Rhomboide EFGH, cum diameter AC, EH eos bifa- riam diuidat in duo Isoscelia æqualia ABC, ACD, & EHG, EHF, & in Rhomboide ex quatuor conuersis con- stituto diameter recta etiam IL in duo semitriangula æqua- lia diuidit ex oppositis angulis ducta.

G 2

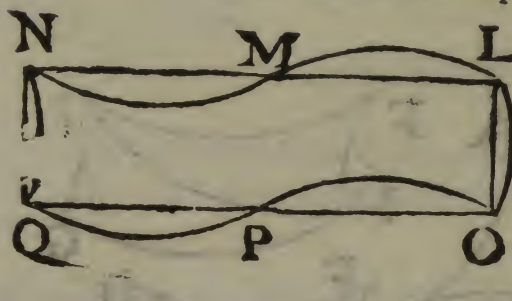
Altera



Altera parte curuilinea, &  
semicuruilinea qua-  
drare. Prop. 10.

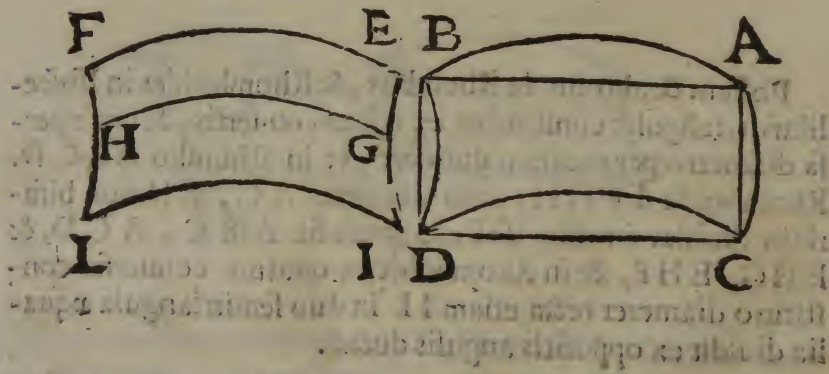


**A** Altera parte longiora quadra-  
bis omnia, vt quadrata, duo-  
bus semper portionibus oppositis  
ablatis, & repositis, vt in ABCD.



Erit altera spe-  
cies altera parte lon-  
gioris curuilinei L  
NOQ demptis sci-  
licet tribus portio-  
nibus LM, PQ, OL,  
repositis MN, OP,  
QN, quadrabitur.

*Corollarium.*

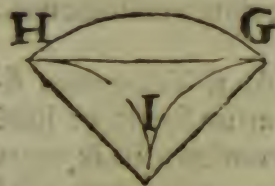


**A**T reliquas species diuides non dimetiente ex angulo  
ad angulum ducta, sed per medium vtrinq. latera pa-  
rallela, vt in EFIL dimetiens GH.

Pe-

## Peleces quadrare. Prop. 11.

**P**ossunt peleces multifariam variare ex varijs circularum circumferentijs, & primo ex partibus, cuius partes circumferentiæ dimidij circuli  $ABC$ , aliæ duæ partes ex duabus quartis eiusdem circuli  $AED$ ,  $DFC$ , vt demptis illis, his repositis, rectilineum quadratum peleci æquale erit.

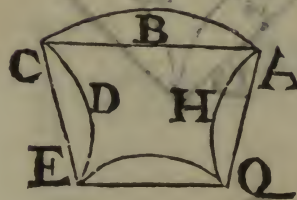


Potest & ex duplicis circumferentijs constitui, vt sit  $GH$  quarta dupli, duæ vero quartæ subdupli  $GI$ ,  $IH$ , quæ additæ rependent ablatum  $GH$ , eodem modo ex quadrupla eueniet. Peleci ex inæqualibus, sed eisdem circumferentijs, & varijs, vt Peleci  $GEABCFDH$  quadranda portio  $ABC$  sit æqualis  $GHD$ , &  $DFC$ ,  $GEA$ , demantur  $ABC$ , reponantur  $GHD$ ,  $DFC$ , & erit quadrilaterum rectilineum  $ACGD$  æquale supradictæ Peleci.

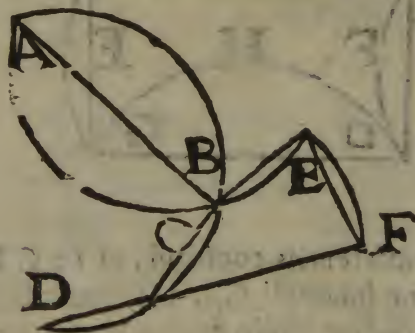
Trape-



Trapezia curuilinea ex æqualibus, & inæqualibus circumferentijs constituta quadrare. Prop. 12.



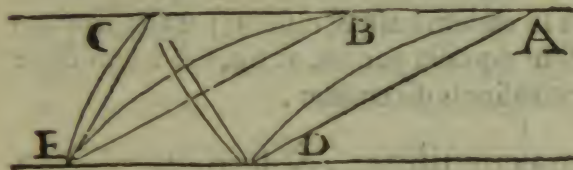
**S**int Trapezia curuilinea ex quatuor, vel pluribus circumferentijs constituta, vel omnibus inæqualibus, vel tribus, aut duobus, dummodo inter eas ita conueniant, vt tres, duæ, aut plures possint, quantum vna, aut aliæ: nam sit portio ABC tripla, & sint tres æquales AHQ, QFE, EDC, dematur maior, addantur tres minimæ, & cœquabitur rectilineum curuilineo.



At si Trapezium figuratum fuerit, vt iisdē circumferentijs, & æqualibus constituatur, sed cū alterū altero longius sit, & quantum in altero deficit in altero superfit, minus addatur superfluo, & fiat æqualis compensatio. AB duæ portiones demantur, addantur duobus alijs BC, CD, & quia pars EF superabit, deficit vero EB, huic addatur illius vice, sic rectilineum BEFDCB curuilineo æquabitur.

Trian-

Triangulum Iſoſcele curvilineum, & parallelogrammum ſemicurvilineum in eadem baſi conſtituta, & eiſdem parallelis, parallelogrammum triangulum duplum erit, & rectilineis æqualia erunt. Prop. 13.



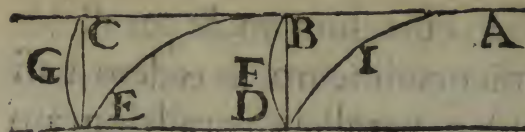
**S** It triangulum Iſoſcele ſemicurvilineū DCE, & parallelogrammum ſemicurvilineum ABDE in eiſdem parallelis ACDE dico pa-

rallelogrammum in eadem baſi, & eiſdem circumferentijs conſtitutum eſſe triangulo duplum. Quoniam portio DC ipſi CE æqualis, dematur EC, addatur DC, erit triangulum rectilineum DCE curvilineo æquale. Et quia portio AD ipſi BE æqualis, dematur BE, addatur DA erit rectilineum parallelogrammum ABDE ſemicurvilineo æquale, ſed rectilineum ABDE triangulo DCE duplum eſt; quia in eadem baſi, & eiſdem parallelis conſtituta per 41. 1. Eucl. ergo parallelogrammum rectilineum curvilineo triangulo duplum.

Parallelogramma ſemicurvilinea in eadem baſi, & æquidistantibus circumferentijs conſtituta, & inter parallelas æqualia ſunt. Prop. 14.

**S** Int duo parallelogramma BFDCE, & AIDBHE in eadem baſi DE, & in eiſdem parallelis rectis AC, DE



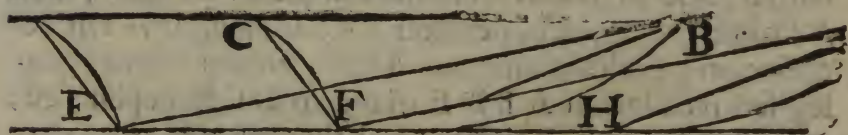


DE constituta, dico  
inuicem esse æqualia:  
trahantur rectæ AD,  
BE, EC, quia portio  
BFD est equalis CGE  
dematur CGE, repo-

natur BFD, rectangulum parallelogrammum curuilineo  
æquale. Idem dicendum de altero parallelogrammo AIDB-  
HE curuilineo æquale est rectilineo ADBF, & quia paralle-  
logramma rectilinea in eadem basi, & eisdem parallelis consti-  
tuta ad inuicem sunt æqualia per 36. 1. Euclid. Idem & de  
parallelogrammis curuilineis dicendum.

Parallelogramma curuilinea, & semicuruilinea  
cum æqualibus basibus, & eisdem circum-  
ferentijs, & eisdem parallelis consti-  
tuta inuicem sunt æqualia.

Prop. 15.



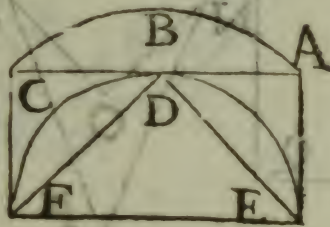
**S**Int duo parallelogramma semicuruilinea, & æquidistan-  
tibus circumferentijs AH, GB, & CF, DE, & æqualibus  
basibus constituta FE, GH, & in eisdem parallelis AD, HE  
dico esse inuicem æqualia, trahantur rectæ AH, BG, CF, DE,  
AF, BE, quia AH portio æqualis est BG, dempta AH re-  
posita BG, erit rectilineum AHBG curuilineo æquale, & idem  
de alio CFDE, sed rectilineum AHBG curuilineo æquale, &  
idem de alio CFDE, sed rectilineum CFDE in eadem basi  
cum rectilineo ABFE, & ABFE in eadem cum AHBG, ergo  
inuicem æqualia per 26. 1. Euclid. ergo &c.

Paral-

Parallelogramma semicurvilinea in eisdem parallelis constituta, & ex diuersis circumferentijs videlicet duplis dari possunt rectilineis æqualia.

Prop. 16.

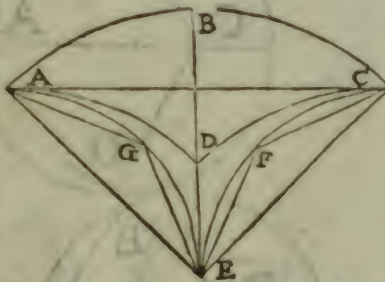
**S**it parallelogrammum semicurvilineum  $EABCFDE$ , & sit portio  $CBA$  dupli, subdupli autem semicirculus  $FD$ , dico quadrari posse, trahatur linea  $CA$ , &  $FD$ ,  $DE$ , quia duæ portiones subdupli  $FD$ ,  $DE$  valent quantum vna subdupli  $CBA$ , dematur  $CBA$ , reponantur duæ subdupli  $FD$ ,  $DE$ , rectilineum  $EACFD$  valet quantum semicurvilineum.



Triangulum tricuspidale quadrare.

Prop. 17.

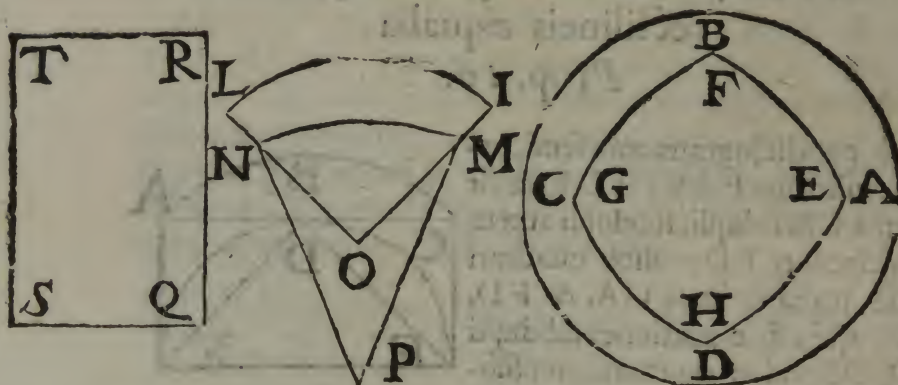
**E**X quarta nostri secundi representetur figura  $ABCFEG$ , à medio  $AC$  trahatur linea  $BE$ , & linea  $EB$  signetur in linea  $BE$ , linea  $CD$  ex eadem dupli circumferentia idem ex altera parte, dico triangulum tricuspidale  $ADCFEGA$  quadrari posse, diuidantur  $EC$ ,  $EA$  bifariâ in  $FG$ , & trahantur lineæ  $EF$ ,  $FE$ ,  $EG$ ,  $GA$ , sic, & lineæ  $DC$ ,  $DA$ , quia  $DC$  portio est octaua pars sui circuli, portiones  $EF$ ,  $GC$  duæ octauæ subdupli æquipollent vnam dupli, sic eam demendo, has addendo,





do, trapezium EFCD rectilineum respondet curvilineo EFCD, idem de alia parte dicendum, & multifariam potest euenire.

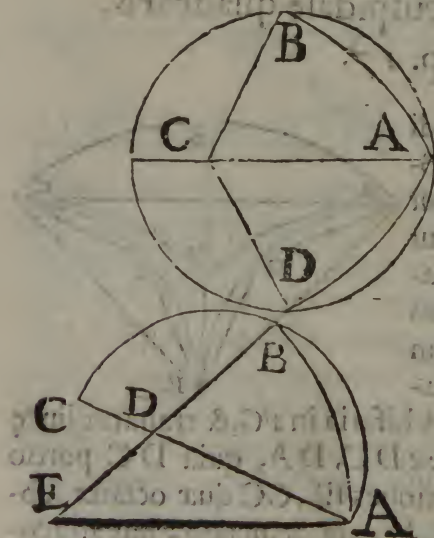
Coronas quadrare. Prop. 18.



**S**it quadranda Corona ABCDEFGH, pono eius quadrantem ILO, & ibi comparem octauā partem circuli dupli MNE, tollatur cōmune MNO, remānet tricuspīdale triangulū quadrilaterū MONP

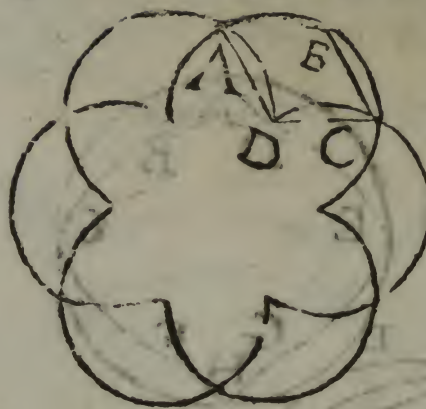
æquale quartę parti coronę IMLN, quæ æqualis AB EF, quadruplicetur cuspidale triangulū, & erit ipsius area QRST æqualis coronę propositę.

Eodem modo quarta pars semicirculi dupli ABE absorbit dimidium circulū ABC subduplum, tollatur cōmune triangulū ABD, remānet ADE triangulū rectilineum æquale Lunę AB, & circuli BDC, quod duplato æquipollet coronę ABCD. Sit



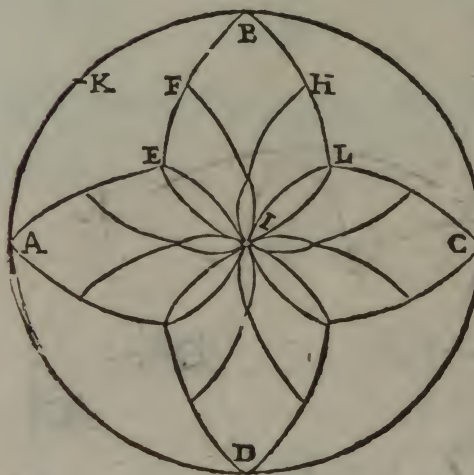






& sit  $AD$  octava pars sui circuli, &  $AE$  sui circuli, duæ igitur portiones  $AD$ ,  $DE$  æquivalent vni maiori: octo igitur eiusmodi triangula respondent oppositæ coronæ.

Corona femiquadranda.



**E**Sto circulus  $ABCD$  cuius quadrans  $ABI$ , triangulum  $ABE$  notum est, quia  $AE$ ,  $EB$  æquales sunt circumferentiæ  $AK$ ,  $KB$ , reliqua pars quarta  $EFBHLI$ , quia  $BF$ ,  $BH$  æquales sunt  $GF$ ,  $GH$ , tolle  $FB$ ,  $BH$ , repone  $GF$ ,  $GH$  erit nota pars  $FB$ ,  $HG$ , reliqua pars  $EF$ ,  $IG$  nota est, quia æqualis  $FE$ ,

$EI$ , tolle, & repone nota erit pars illa, remanent ergo 4. portiones  $IG$ .

Volutas omnifarias quadrare. Prop. 19.

**E**St voluta figuræ species in coclæ modum sinuata, cuius ambiens perpetuo flexu ducitur binis in se quoddammodo-

dammodo recuruis, & refractis lineis.

Propositum ergo sit quadrare volutam BED GIFCHLMA scio hanc volutam octauam esse partem volutæ circuli NOPQRS, & omnes circuli volutæ non capiunt nisi octogoni aream, ergo vnaquæque octauam partem com-



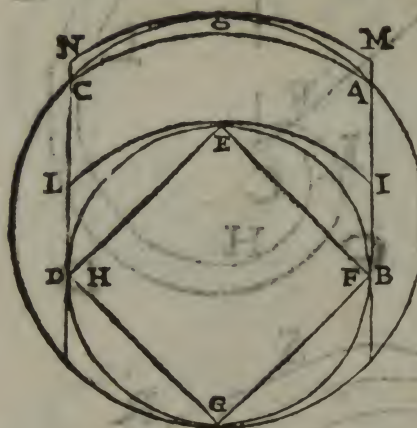
plectitur: vna igitur volutæ pars est BED GIFCHLMA est octaua circuli pars, & octaua circuli pars est triangulum ABC, ergo tota proposita voluta quadrata triangulo meretur. Possumus, & hoc modo cyssoidem triangulum etiam quadrare, quod vidimus in 5. propositione huius.

Curui-

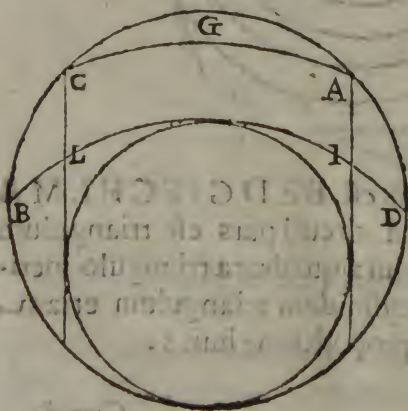


## Curvilinea triangula aliqua quadrare.

Prop. 20.



It circulus duplus AM-  
 NC, subduplus vero  
 EFGH, seq. in puncto con-  
 tingant G absceindantur à  
 duplo duæ portiones qua-  
 drati CN, AM, & à subdu-  
 plo quatuor quadrati EF,  
 FG, GH, HE remanent va-  
 cua CDEBA, BMG, GNH  
 æqualia quadrato EF, GH  
 puncto E ducatur parallela  
 CA, & sit LO quadrangu-  
 lum CLA I notum est, remanent quatuor triangula LDE,  
 EIB, AMO, CNO puncto dupli circumferentia ducatur MN,  
 & ex punctis CA alia parallela eiusdem circumferentiæ CA,  
 dico lunulam COA quadrari posse triangula NCO, OAM  
 nota sunt, quadrangulum NCMA notum, quia ex paralle-  
 lis circumferentijs, à quo si triangula subducantur COA  
 lunula nota remanet.



Quomodo lunæ cor-  
 nicula quadrari pos-  
 sint. Prop. 21.

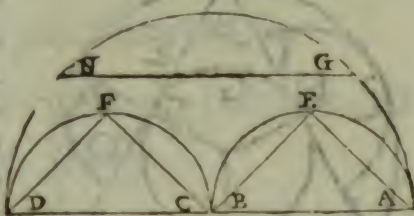
R Emaneant superior de-  
 scriptio, & in semi-  
 circulo BGD describatur  
 dupli circumferentia BLID  
 à punctis BD, lunula igitur  
 BG-

BGA DILB nota est, lunula parua nota est CGA, quadrangulum CLAI notum etiam ex anteriori, remanent ergo corniculi CBL, AID etiam noti.

Trapezia multa curvilinea quadrare.

Prop. 22.

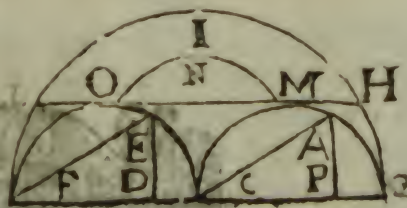
**P**ossumus quadrare Trapezium AGNDFCBEA, quia semicirculus AGND est quadruplus AEB per 20. nostri, duo semicirculi AEB, CFD valent quantum arbilon AGNDCBA, dematur portio GN quarta circuli pars, & 4. portiones AE, EB, CF, FD, remanent duo triangula rectilinea AEB, CFD aequalia trapezio curvilineo iam dicto.



Eadem ratio erit in trigono. sit trigoni portio LIN, & duo circuli FED, CAG, a quibus due portiones FE, CA, & due dimidia ED, AG, quae vnam integrant, altera erit OIM, arbilon FLINGMCO valet duos semicirculos, a quibus si tres dempseris portiones, tres item ab arbilone, vacua FLO, OKM, MNC, NMP, TOI valent duo trigona FED, CAB.

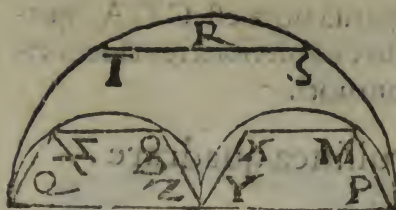


Eadem ratio erit in exagono, & trigono, nam in trigono in circulo GHILF, duo trigona APC, DEF, aequipollent vacuis GHM, MOC, OLF, HILONM, & in exagono PSRTQ,

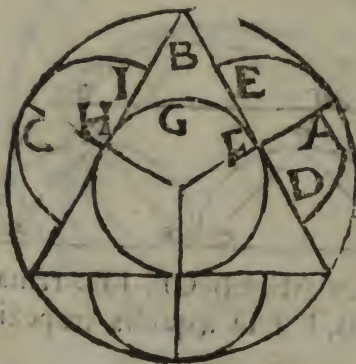


duo

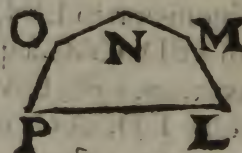
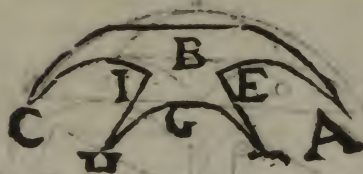




duo semihexagona  $P M X Y$ ,  
&  $Z 8 \& Q$ , valet vacuum  
 $P S T Q Y Z$ , in linea  $H L$ ,  
tangens circumferentiam cir-  
culorum est latus trigoni æqui-  
lateri per 12. 13. Eucl.



Circulus  $A B C$ , est qua-  
druplus  $D A E$ , ergo pars ter-  
tia circuli  $A B C$ , quæ est  
 $A B C$ , est unius circuli, &  
tertiæ partis, pars eius tertia  
est  $F G H$ , reliquum ergo erit  
corona  $A B C H G F$  dema-  
tur duo quadrantes circuli  
 $A E F$ ,  $C H I$ , remanet va-  
cuum  $A E F G H I C B A$ , quan-  
titatis dimidij circuli, & quia



octava pars circuli maioris, valet quatuor minoris dematur  
portio  $B$  ex maiori, & 4. 8. ex minori  $L M$ ,  $M N$ ,  $N O$ ,  $O P$ ,  
ergo rectilineam  $L M N O P$ , valet trapezium  $A E F G H I C B A$ .



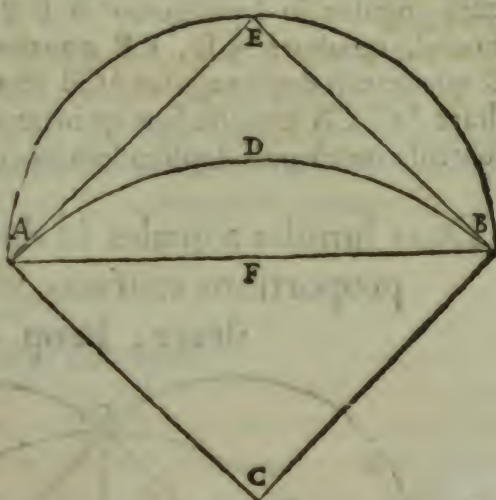
65

IO. BAPT. PORTÆ  
NEAPOLITANI  
ELEMENTORVM CVRVILINEORVM  
Liber Tertius.

*In quo de Circuli quadratura agitur.*

Lunulam ex dupla, & subdupla proportionē  
quadrare. Prop. 1.

**D**Escripto duplis  
circuli quadrā-  
te his caracteribus  
distinguat ADBC,  
cuius subtensam AB,  
scinde bifariam, & pun-  
ctus scissionis ad am-  
plum medium F chara-  
cterē sortiatur, in quo  
circini pede infixo ex  
FA interuallo circum  
ducto semiambitum  
subdupli AEB ducito,  
aio triangulum recti-  
lineum ABC interceptæ lunulæ areæ AEBC æqualem esse.  
In medio periferiæ subdupli E punctus instituendus, & ab  
utraque circuli extremitate A B lineæ excurrant vsque ad E,  
ibique mutuo concurrant, quia dupli portio ADB quarta  
sui circuli pars valet quantum duæ subdupli portiones AE,  
EB, etiam sui circuli pars quarta (per 20. primi nostri) ideo  
sub-





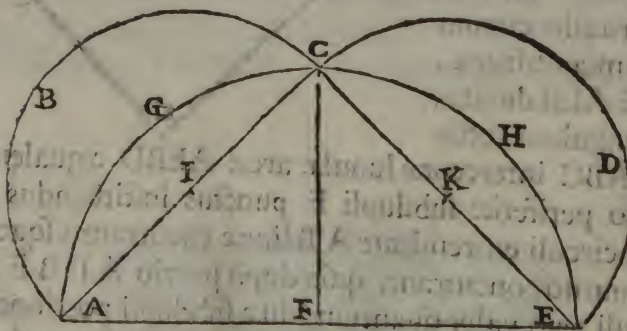
subductis portionibus A E, E B apposita A D B (per primum axioma nostri secundi libri) triangulum ABC valet quantum lunula AEBD, quod erat demonstrandum.

Hypocrates hoc aliter probat in primo Physicorum Aristotelis, quia quadrans dupli ADBC valet quantum semicirculus subdupli AEB, abscissa portione communi ADB, quæ inter vtrumque interiecta est, remanet trimetrum A B C æquale lunulæ AEBD, quadrandæ.

### *Confectarium.*

**E**X hoc circumferentia dupli transibit semper per extremitates diametri subdupli, quod in alijs non euenit; quia angulus in semicirculo A E B rectus est (per 31. 3. Euclid.) quadrata A E, E B æqualia sunt quadrato AB, & quadrantis dupli angulus ACB etiam rectus est, ergo quadrata AC, CB æqualia sunt quadrato AB, ob id recta linea subtenfa quadrantis dupli eadem est cum diametro subdupli.

Duas lunulas æquales in dupla, & subdupla proportionem exaratas seorsim quadrare. Prop. 2.



Eodem modo hoc commodissime absoluemus. Esto circulus

culus dupli ACE, & diametri extremitatibus AE binæ lineæ in mediū circuli anfractum excurrant, & ubi mutuo contactu angulum efficiunt, illic C litera exaretur. Subten-  
 sa AC, CE in medio ductu præcidantur, præfixis literis IK, mox assumpto circino ad rem commodè proferendam, pede vno in I infixo subistente IA interuallo linea circumducatur vsque donec ad alteram extremitatem C perueniat, inuariatoq. circini pede K puncto compari linearum perscriptio-  
 ne circuli semianfractum designet ADE. Hoc peracto pun-  
 cto F lineæ in plano iacentis perpendicularis extollatur, quæ in cuspide curtiaturam C contingat, aio lunulas ABGC, CDEH, æquales esse trimetro ACE. Quoniam circulus ACE duplex est ABC, CDE, ergo semicirculus ABC, CDE æquales sunt semicirculo ACE, amputentur duæ communes portiones AGC, CHE, residua rectilinea trian-  
 gula ACF, FCE æqualia sunt lunulis ABGC, CDEH vel modo, quo supra præcepimus triangulum dupli AGCF æquale est semicirculo ABC, & triangulum FCHE æquale semicirculo CDE, subductis communibus portionibus AGC, CHE, triangulum ACE, est æquale duobus lunulis ABGC, CDEH.

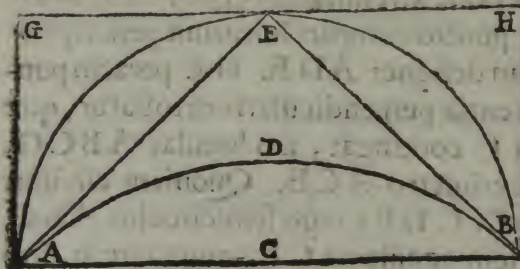
### *Conseclarium.*

**A**Ntequam ultra progrediar consentaneum duxi adno-  
 tandum angulum rectum ACE bifariam dissectum in C ex (per 8. 6. Euclid.) EC, ad CA rationem habet vt EF ad FA, & sic triangulum CFE ad triangulum CAF (per primum 6. Euclid.) & quia æqualia sunt, lunulæ quoque æquales sunt.



Vacans spacium, quod intra figuras omnes  
notas interuenerit quadrare.

Prop. 3.



**R**ecta linea AB  
dirigenda est,  
& ab eius umbilici  
medio puncto C se-  
miorbis circumdu-  
cendus est AEB,  
completo semiorbi  
ACB, lunula com-  
plenda est more

AEBD, mox laterales lineæ erigendæ ab extremitatibus  
AB sunt, & medio eius puncto E superior linea exaretur  
ipsi AB æquidistant, ut vltro, citroq. semicirculum tangant,  
etiam hisce lateribus parallelogrammum expriment AGHB,  
demum ab extremitatibus AB, medio puncto E transuersas  
lineas sortiatur AE, EB. Quoniam parallelogrammum  
GABH semicirculum continet, & est sui quadrati dimidium,  
semicirculus lunulam continet AEBD, & lunula AEBD suo  
triangulo AEB æqualis est, & triangulum AEB sui paralle-  
logrammi dimidium est, ergo interceptæ areolæ AGE, EHB,  
ADB, quæ lunulam AB ambiunt, æquales sunt ipsi lunulæ  
areolæ, ergo sui amplexantis parallelogrammi dimidium  
sunt.

*Confectarium:*

**E**X hoc perspicuum est lunulam sui quadrati partem esse  
quartam; nam si lunula sui obsepientis parallelo-  
grammi dimidium est, & parallelogrammum sui quadrati  
dimi-

dimidium; igitur lunula sui quadrati dimidium erit.

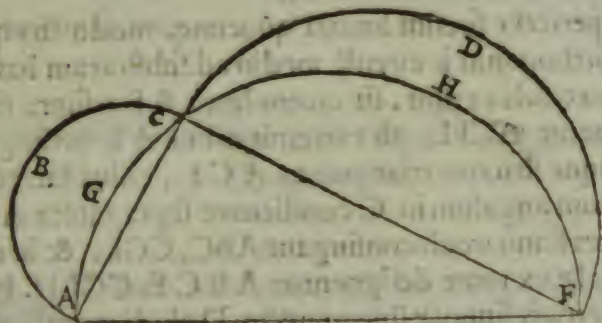
Vel si cuiusunque figuræ notæ vacua quadrare velimus modo supra cognito notæ figura circumclaudatur, quam si à notæ subtrahes, optato poteris ex secundo axioma (secundi nostri) sit gratia exempli pelecis



HEIGOF sapienda suo parallelogrammo ABCD, quam ab ipso seduces, sic inclusæ areæ HAE, EBI, IGOD, OCHF residuum innotescet.

Duas quascunque lunulas inæquales in semicirculo fitas simul quadrare.

Prop. 4.



**T** Riangulum rectilineum in semicirculari linea definiri debet, quod tribus notis distinximus ACE, supra eius latera semicirculi incubabunt ABC, CDE, quibus congruens area adinuenienda est, inquam angulus ACE in semicirculo rectus est, & bini semicirculi ABC, CDE æquales sunt semicirculo AGCHE ex eis, quæ supra habita sunt, reiectis communibus portionibus ACC, CHE relictæ semilunulæ ABCG, CDEH residuo triangulo ACE rectilineo æquiparantur.

At





fixo, altero vago in A collocato sinuosa linea ducatur vsque ad L. Quoniam duæ lunulæ perfectæ ABCE, CGLH æquales sunt vni lunulæ ASL (ex prima huius) & lunula AGDFLS est æqualis triangulo AKL, quod idem est cum triangulo ADL.

*Confectarium.*

EX hoc animaduertendum imperfectæ lunulæ quanto magis à semicirculi vertice declinant, tanto minores fieri, vt in triangulo ACL videre est, quod triangulo ADL minus est, & qui defectum conspiceret quæsierit, triangulum ACM à triangulo MDL subducat hoc modo, à linea MD, linea MC præcidat, & à linea ML lineam MA obtruncet, & lineam MN ducat, triangulum MIN æquale erit ACM, reliquum triangulum IDN erit quantitas lunulæ IDML, dempla lunula AEC.

Vacua inter lunulas intermissa quadrare.

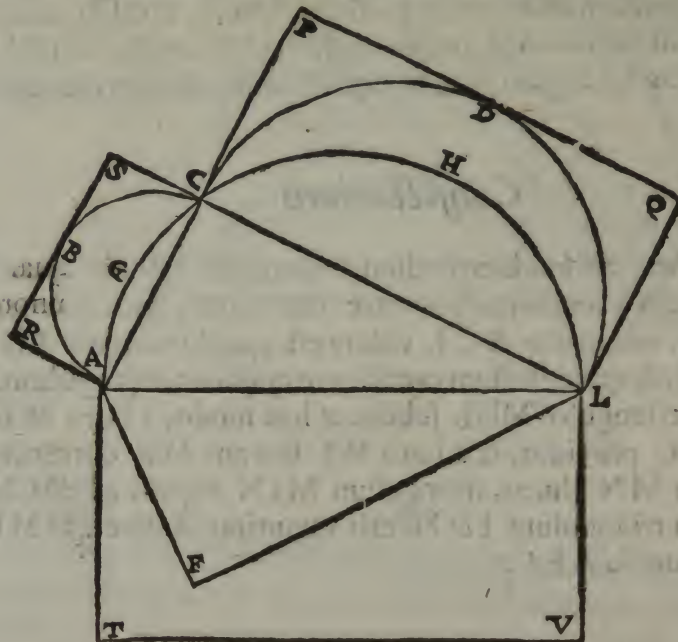
Prop. 5.



T vero interuenientia vacua circa lunulas si quadrare quæsieris, ita quadrabis. Esto minor lunula ABCE, maior CDLH imperamus semicircuilinea triangula inania inter illas, in rectilineas figuras reddere scilicet CPD, DQL, CHL, ARB, BSC, AGC, circumscribantur parallelogrammata tangentialia earum ambientes lineas PQCL, ARSC, & fiat alterum parallelogrammum ex binis AL, TV, & sit ALTV, & fiat triangulum ACL æquale AFL (per 31. primi Euclid.) quibus ita dispositis inquam vacuum circa ATVLÆ æquale esse imperatis vacuis.



cuis. Quoniam triangulum  $AF L$  est æquale  $ACL$  ex



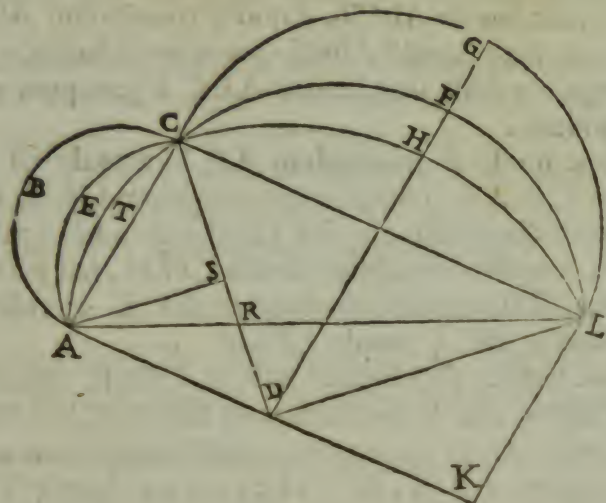
constitutione, & triangulum  $ACL$  est æquale lunulis  $CDLH$ , &  $ABCG$ , ergo si triangulum  $AFL$  à parallelogrammo  $ATLV$  abstuleris, reliquum vacuum  $ATVLF$  erit æquale interiectis vacuis iam recensitis.

Duas lunulas inæquales in semicirculi ambitu  
descriptas seorsum quadrare.

Prop. 6.



STO rectangulum triangulum  $ACL$ , cuius  
porrectius latus  $CL$  sit duplum exilioris  $AC$ ,  
& circumferantur lunule ex more, quibus adij-  
ce suas literas indices  $CGLF$ , &  $ABCT$ ,  
mox parallelogrammum constituatur ex late-  
ribus



ribus AC, CL, & sit ACKL, & fiat quadrans circuli CDLH,  
 & subdupli AECS, nos rationem reddituri, lineam CR trian-  
 gulum ACL partiri taliter, vt anguli compares mutuo cor-  
 respondentes, & æquales sint, vt ACR par sit RDL & trian-  
 gulum ACR par sit lunulæ ABCT, & triangulum CRL ipsi  
 CGLF. Quoniam lunulæ CGLF, & ABCT pares sunt trian-  
 gulo ACL quarta huius nostri adiuuante, & triangulo ACL  
 par trimetrum CDL, quoniam vtrique sui parallelo-  
 grammi dimidium est (vt figura quarta huius demonstratum  
 est) ergo triangulum CDL est duabus præsignatis iam lunu-  
 lis CGLF, & ABCT æquale, sed triangulum CDL est æquale  
 lunulæ CGLH, ergo lunula CGLH est æqualis CGLF, &  
 ABCT subducatur semilunula CGLF, vtpotè vtrique com-  
 munis, remanet sublunula CELH æqualis ABCT, & quem-  
 admodum triangulum CDL æquale ACL, subducatur com-  
 mune CRL, reliquum triangulum RDL reliquo triangulo  
 ACR æquale, lunularū partium representantia, sequitur trian-  
 gulum

K

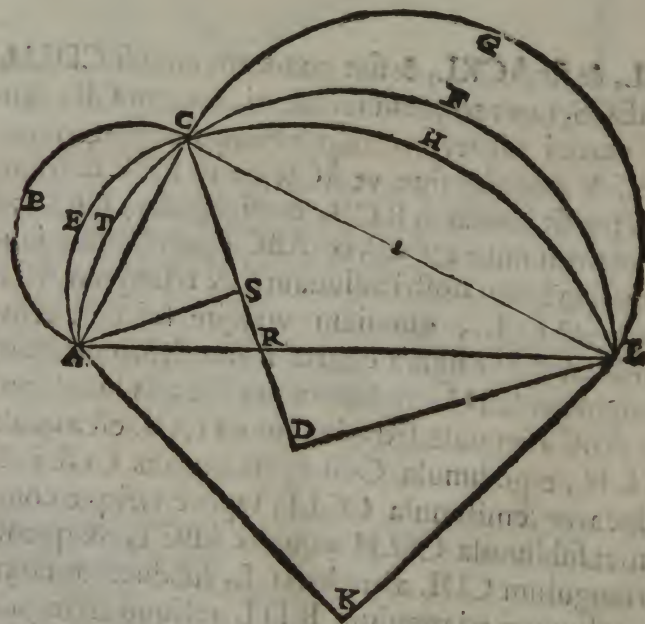
gulum



gulum  $RDL$  esse æquale sublunulæ  $CFLH$ , & triangulum  $ACR$  æquale lunulæ  $ABCT$ , à quo si triangulum  $ACS$  subtrahatur, æquale lunulæ  $ABCE$  (per primam huius) remanet subtriangulum  $ASR$  imæ lunulæ  $AECT$  par, quod erat demonstrandum.

Vel hoc modo si triangulum  $ACL$  æquale est lunulis  $ABCT$ ,  $CGLF$ , compleatur triangulum  $CRL$ , & compleatur perfecta lunula, quæ sit  $CGLH$ , ergo addita pars trianguli  $RDL$  æqualis erit additæ lunulæ  $CFLH$ , sed pars trianguli addita  $RDL$  æqualis est triangulo  $ACR$ , vt vidimus, & par est lunula  $ABCT$  lunulæ  $CFLH$ .

Vel hoc modo. Tres lunulæ  $ABCE$ ,  $CGLF$ ,  $AECT$  sunt æquales trigono  $ACL$ , & duæ lunulæ  $CGLF$ ,  $CFLH$  trigono  $CDL$ , ergo omnes quatuor iam dictæ lunulæ sunt æquales duobus trigonis  $ACR$ , &  $CDL$ , sed tres lunulæ  $ABCE$ ,

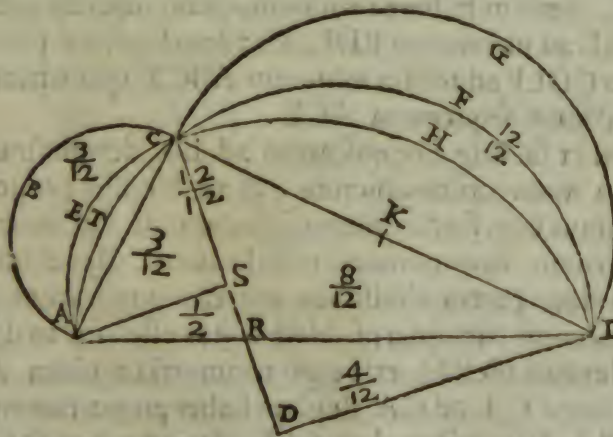


$CGLF$ ,  $LFCH$  sunt æquales trigono  $ALK$ , ergo reliqua lunu-

linula A E C T est æqualis trigono A S R, quod querebamus.

Vel tres lunulę  $ABCE$ ,  $AECT$ ,  $CGFL$  æquales sunt tri-  
gono  $ACL$ , & trigonum  $CDL$  æquale lunulę  $CGLH$ , tolle  
triangulum  $CDL$  æquale iam dictę lunulę  $CGLH$ , reliquum  
triangulum  $ACR$  lunulis  $ABCE$ ,  $AECT$  æquale est, tolle lu-  
nulam perfectam  $ABCE$  sub lunula  $AECT$  sub triangulo  
 $ASR$  æqualis erit, quod erat demonstrandum.

Trianguli in circulo descripti angulo per medium discisso, & lunulis à circulo medio diuisis, proportio partis maioris trianguli ad minorem, est sicut superior pars maioris lunulæ ad inferiorem, & superior eadem pars ad minorem lunulam, & superior pars trianguli ad inferiorem sequitur eam suarum lunularum. Prop. 7.



Præusquam ad diuersarum partium rationem lunularum  
K 2 descen-



descendamus, admonitione dignum censemus, quod cum ab æqualitate duarum lunularum descendimus, quam (in secunda parte) vidimus, quantum maior crescit, tantum altera decrescit, & ex alterius defectione altera augmentū suscipit, & circulus ille, qui per medium vtriusque percurrit, à maiori semicirculo subripit, & minori addit, sed id non temerè, sed certo se superant excessu, vt ratio maioris superioris lunule ad inferiorem eadem sit, quam maioris superioris trianguli pars ad inferiorem, & ratio maioris superioris lunule ad totam minorem, vt ratio partis trianguli anterioris ad posteriorem eandem sequuntur analogam, & vtraque vtriusq; rationem sequitur, vt exemplis patebit. Triangulum rectangulum strue, cuius angulis appinges literas ACL, productius latus CL in duas partes, angustius in vnam partiri. In puncto bifariæ scissionis lateris CL signa K, ex quo, & interuallo CK circinationis arcus exaretur, cui suas indices literas applicabis CGL, eodemq. ordine signa AC suum arcum delineā, ABC, mox triangulum maioris circuli CGL constitues, & sit CDL, & minoris ABCE, sit ACS supra succumbentem omnium basim AL, medius circulus flectatur ACFL, dico lunulam superiorem maioris circuli CGLF ad suam inferiorem CFLH, eandem habere rationem, quam superior pars trianguli CRL ad inferiorem RDL, & eadem superior pars lunule maioris CGLF ad totam minorem ABCT, quam triangulum CRL ad suum sequentem ACR.

Quod vt facilius cognoscamus ad hoc demonstrandum adhibeo numerorum officium, & vt facilius proportionem obseruemus cum fractis integros numeros in fractiones soluamus, vt vnum denominatorem habeamus. Quoniam lineam CL in duas partes diuisimus erit eius quadratum quatuor partium, cuius pars quarta, idest vnitas est, hanc in duodecimas soluemus, idest  $\frac{1}{12}$ , erit ergo totum triangulum ACL  $\frac{1}{12}$ , & quia linea CL ad CA, duplam habet proportionem, ita RL ad RA, & sic triangulum CRL ad triangulum CAR, ergo

trian-





maior, ABC minor. Inde perfectæ lunulæ ex ordine subfi-  
gnentur, idest trianguli CDL, CGLH, trianguli ACS,  
ABCE.

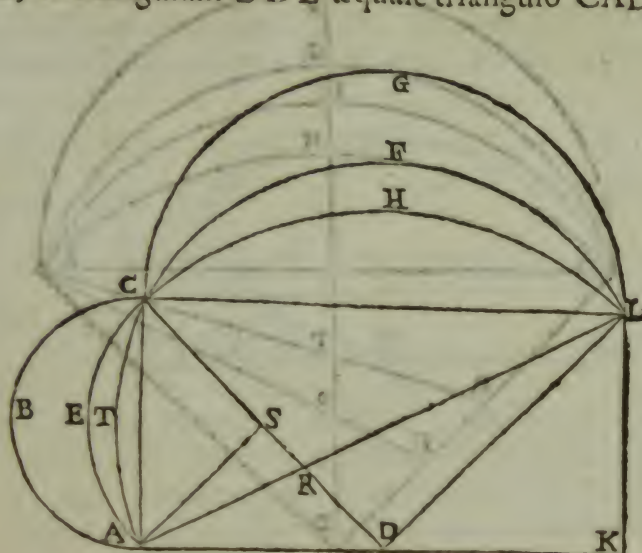
Triangulum ACL, quia linea est trium partium, area erit  $\frac{1}{2}$ , proportio CRL ad ACR est tripla, ob id triangulum CRL erit  $\frac{1}{3}$ , & triangulum ACR  $\frac{2}{3}$ , triangulum CDE latus trium est partium, quadrans est nouem partium, cuius quarta pars est  $2\frac{1}{4}$ , idest  $\frac{1}{4}$  supra sunt trianguli CRL  $\frac{1}{3}$ , ergo triangulum RDL erit  $\frac{1}{3}$ , tres lunulæ ABCE, AECT, CGLF est  $\frac{1}{2}$  superior maior ad duas minores est tripla, ergo lunula superior est  $\frac{1}{3}$ , & duæ lunulæ ABCE, AECT est  $\frac{2}{3}$ , sed lunula perfecta est  $\frac{2}{3}$ , ergo superius triangulum CAS  $\frac{1}{3}$  subditum reliqui erit  $\frac{1}{3}$  & sublunula reliqua  $\frac{1}{3}$  inferior maior lunula, vt compleat numerum  $\frac{1}{2}$  erit  $\frac{1}{3}$ , ergo proportio lunula superior ad inferiorem, idest CGLF ad CFLH sicut triangulum CRL ad RDL, & lunula EGLF ad lunulas ABCT, & superior lunula minor ad inferiorem, vt triangulum ACS ad triangulum ASR.

Datam maioris lunulam circuli ita secare, vt  
eius sublunula minori lunulæ, & sub-  
lunulæ par sit. Prop. 8.



Vamquam in supra commonitis idem indicaue-  
rimus, vberioris tamen doctrinæ gratia exem-  
plum in hunc modum absoluemus. in hanc ra-  
tionem triangulum orthogonium eligendum  
est ACL, cuius porrectius latus CL bifariam  
dispersemus, supra lunulas ex more constituemus, & medium  
circulum ATCFL, & eis triangula subijciemus CDL, ACR,  
mox datis lineis AC, CL parallelogrammum constituatur  
CALK, inquam sublunulam CFLH, lunulis ABCT aqua-  
lem esse. Quoniam lunula CGLH æqualis est suo triangu-  
lo

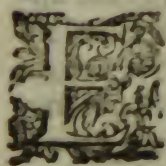
lo CDL, & triangulum CDL æquale triangulo CAL, quia



utrumque est æquale suo parallelogrammo dimidio CALK, & triangulum ACL est æquale lunulæ CGLF, & lunulis ABCT, ergo lunula CFLH est æqualis lunulis ABCE, & AECT, tollatur de medio lunula CGLF, remanebit sublunula CFLH æqualis ABCT, quod erat discutiendum.

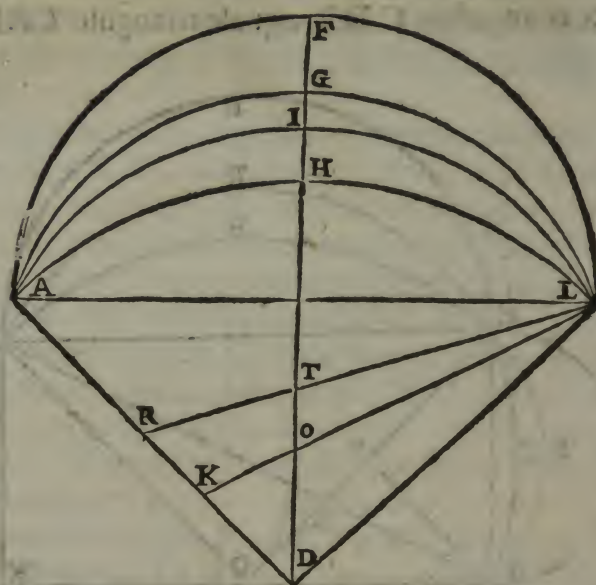
Datam dupli, & subdupli circuli lunulam ad imperatas partes discindere.

Prop. 9.



Sto dupli, & subdupli multifariam scindenda lunula AFLH, cuius subtenfa AL, sui compar triangulum inferius substituemus ADL à medio inflexi circuli vertice F vsq; ad trianguli aciem infernè positam D recto ductu descendat. In dextro trianguli latere AD futuræ linæ designandæ sunt lunulam

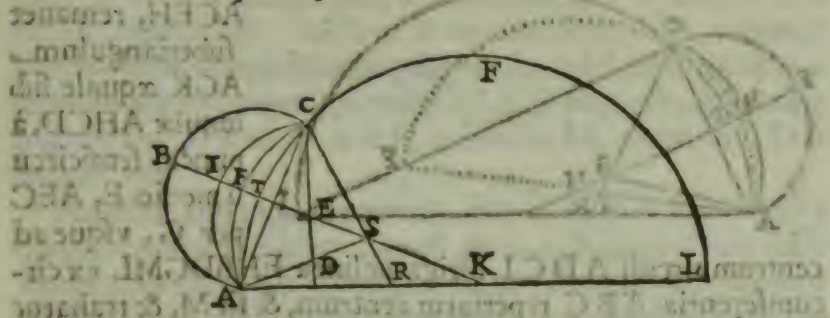




nulam diuisuræ in quocunque partes volueris decem, vel septemdecim, pro nunc bifariam in R dispe scito, & transuer-  
sam ad L deducito, si velis lunulæ mediam partem auferre,  
& vbi se lineæ cum diametri linea decussabunt, fige circini  
pedem fixum, & vagum alterum ad alterutrum diametri  
extremitatem A, vel L, arcum circumflecte AGL, nam  
lunula AFLG erit dimidium lunulæ. Vel si tertiam partem  
vis auferre, sit latera K in laterali tractu tertia pars ab K ad  
L lineam porrigito, & vbi FD lineam secat, pede circini sta-  
bili collocato, ac vago altero ad A extremum, arcum circum-  
duces AIL, & tertiam lunulæ partem AILH à duobus ab-  
scindet. Quoniam triangulum ADL per RL lineam bifariam  
dissectum est, superior trianguli pars ARL per superexaratam  
propositionem superiori lunulæ parti AFLG congruit, & ima  
trianguli pars RDL inæ lunulæ parti correspondet, & vtræq.  
partes sunt, ergo & lunulæ eis pares dissectæ sunt: Idem de  
tertia parte dicendum.

Eadem

Eadem in parua lunula operaberis, nam si bifariam ABC lunulam vs partiri trianguli eius ABS latus, AS bifariam in D, & latus à D quousque ad punctum C perueniat protrahe-

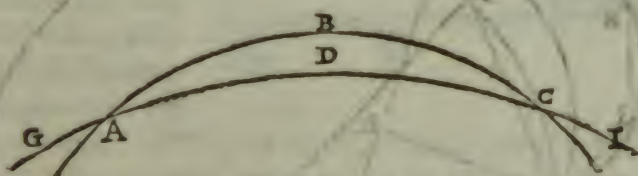


mus, & lineam à B medio semicirculi, punctoq. S, vsque ad circuli ATCFD centrum perueniat, & vbi se intersecabunt puncto E siste circini pedem, & Intervallo EA, extende circulū AIC, & diuisa erit lunula: probatio ex anteriori liquet.

Datam quaecumque lunulam quadrare .

Prop. 10.

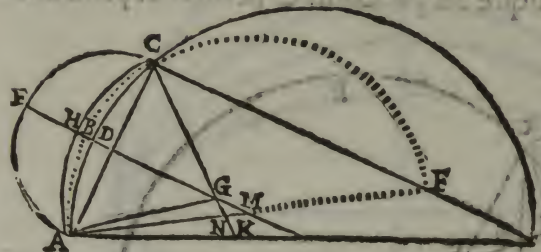
Esto exposita quadranda lunula ABCD cuiuscunque ordinis, cui æquale oportet reperiri rectilineū, maior cir-



culus GADCH, integer circinetur, & sit ADCI porrecto suo diametro AI, & à puncto A superponatur præfata lunula ABC, & compleatur circulus AB, CF cum suo diametro AF, extendaturq. linea à puncto A ad C, & sit AC, discindaturq. rectus angulus ACI per rectam CK, & super basim AC circinetur semicirculus AEC, & puncto G, basi AC, fiat quadrans  
L dupli,



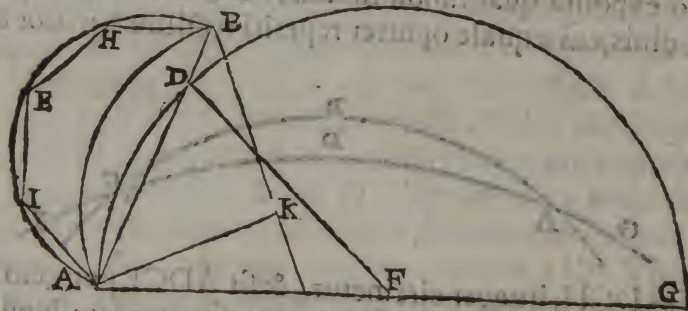
dupli, & sit AHCG, & trahatur AG, GC. Quoniam triangulum A. G. C. est æquale lunulæ



ACEH, remanet subtriangulum AGK æquale sub lunulæ AHCD. à puncto semicirculi medio E, AEC per G, vsque ad

centrum circuli ADCI, dirigatur linea EHBDGML ex circumferentia ABC reperiatur centrum, & sit M, & trahatur AM, vsque ad F diametri finem ex supradictis, si à triangulo AGK, quod lunulam AHCD refert, subducatur AGM, quod representat lunulam AHCB, remanet subtriangulum ANK representans sublunulam ABCD, quod erat edocendum.

Semilunulas ex nota ratione quadrare.  
Prop. xvi.



**I**N sola proportionē dupli, & subdupli accidit, ut semidiameter circuli subdupli sit æqualis subtensæ quadrantis dupli, ut in prima Prop. huius vidimus: ideo in his perfectæ lunulæ, & alijs potius circuli semilunulæ dici possunt.

Esto

est & alio modo cognosci . Portio quadrupli AD  
r portiones subquadrupli, secerur circumferenti  
ariam, & sint portiones AI, IE, EH, HB, ampu  
es AI, IE, EH, HB, reponatur AB, trapezium  
IEHBD notum erit, & sic de cæteris .

*Alio modo .*

Potest & alio modo cognosci. Portio quadrupli AD valet quatuor portiones subquadrupli, secerur circumferentia AEB quadrifariam, & sint portiones AI, IE, EH, HB, amputentur portiones AI, IE, EH, HB, reponatur AB, trapezium rectiligneum AIEHBD notum erit, & sic de ceteris.

*Alio modo.*

A geometric diagram showing a circle with center A. A horizontal line segment GF represents a diameter. Point F is on the right side of the diameter. A point B is on the upper circumference. A line segment AB connects the center to B. A point D is also on the upper circumference, between F and B. A line segment AD connects the center to D. A curved line segment BC connects points B and C. A point C is on the circumference to the right of D. A line segment AC connects the center to C. The diagram illustrates the relationship between the area of a circular segment and a triangle.

Possumus etiam proximè prædicto modo quadrare, quia portio  $AD$ , est dupla ipsius  $ACB$ , diuidatur  $ACB$ , in duas partes  $AC$ ,  $CB$ , demantur hæ duæ portiones  $AC$ ,  $CB$ , reponatur  $AD$ , rectilineum  $ACBD A$ , est æqualæ sublunulæ  $ACBDIA$ .

L 2      Lunu-

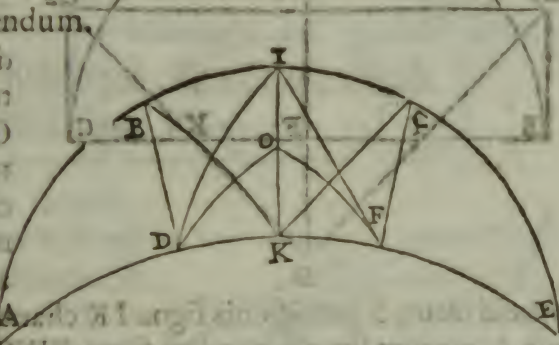
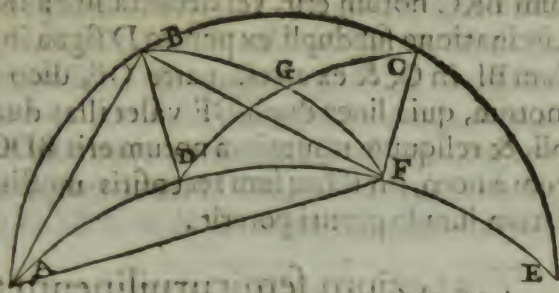






Vel circuli dupli tripartita partitione, curvaturā constituas A D, D F, F E; Item subdupli AB, BC, CE; inuariataq. circuli apertura intra duo puncta B F, DC mutuo inter se intercedentes arcus flostantur BF, DC, & ubi futurus est cōtactus illic pone G, aio triangulum ABF notum esse, quia AF, dupli circumferentia, duobus illis subdupli AB, BF responder; unde reiectis illis, hac reposita portione pensatur. Idem de alia parte DCE dicendum, & triangula BGD, CGF, BGC, DGF nota sunt, sic etiam triangulum BCF, & FCE dicendum.

Remanente adhuc lunule illa tripartita, ne ex puncto D fixo circuli pede altero ad F signa in subduplo æqualē portione in I, mox inuaria ta circuli apertura, qua duplū circulū constituiſti, siste circuli pedem in altera extremitate lunulæ puta E, alterum ad D, & I conuertē, & signa circumferentiam dupli DI, & ab I ad F excutrat recta IF, dico triangulum DIF notum, quia circumferentia dupli DF est æqualis DI ex constitutione, ratione iam sæpius repetita, addendo, & demendo notum erit triangulum DIF, vel intercape dine BK circinetur BK circumferentia dupli, & recta ducatur KC, quia DF est compar BC, & DF æqualis ipsi BK, tolle portionem BC, repone BK triangulum





lum BKC notum erit, vel deducta linea IK, & interuallo BI circinatione subdupli ex puncto D signa in linea IK interual- lum BI in O, & ex altera parte OF, dico triangulum DOB notum, quia linea dupli DF valet illas duas DO, OF subdu- pli, & reliquum trapezium notum erit BDOFC demendo no- tum à noto, similibus iam recensitis modis multipliciter per- latum lunula partiri poterit.

Trapezium semicurvilineum trapezio par extruere? Prop. 73.



Sto dupli qua- drans DGLH, & hinc inde à ter- minis GH trans- uersa linea ducen- da est, quæ curua- turam spectet, & sit GH, mox duo la- tera, quæ anguli fa- ciem in D confir- mant DG, DH, amissim in medio

præcidantur, & præcisionis signa IK characteribus insignian- tur, & prouat linea per eadem signa BIKC ipsi GH paralle- la, & ex punctis G H ad perpendicularum demittantur supra BC lineæ GB, GH, & excurrentes in longum lineam BIKC bifariam diuidat, cuius diuisionis terminus F, & centro F interuallo FB, circumducatur semicirculi forma subdupli BAC, quæ quadrantis circumferentiam in duobus punctis interciderit. Intercisionis puncta totidem literas sortiantur M, N, & uniatur FD. Dico Trapezium semicurvilineum BGMLNHC par esse quadranti DGLH, sed ad demonstra- tionem accingamur. Quoniam FD æquidistat HC, cadit inter



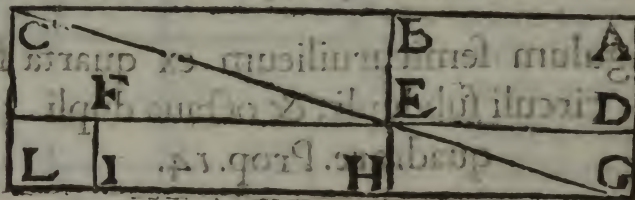




¶ Vel quia lunula æqualis est AGF ipsi ECG ex præcedenti;  
parallelogrammum EBAF est notum, quia ex æqualibus cir-  
cumferentijs; ergo & triangulum CBA notum est, quia vtri-  
que additur commune CAF.

¶ Vel triangulum semicircuilineum, quod paulò antè descri-  
psimus ABCE remaneat, & ducatur lineæ CA portio semidi-  
pli CCA, & sit ABH semiportio dupli, ergo æquales; abscissi-  
sus communis EHGA tollatur, remanet BEA æquale CEH,  
addatur vtrique communis BEC, ergo triangulum rectili-  
neum BHC est æquale triangulo semicircuilineo BCGA.

Notum à noto subtrahere. Prop. 15. E



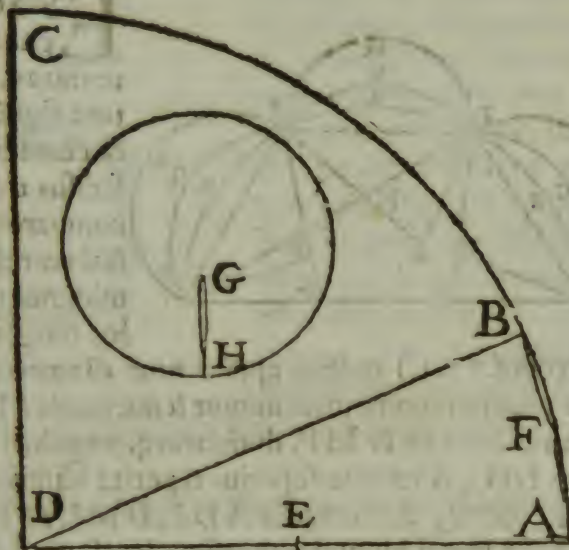
Sed modum, quo notum à noto subtrahatur nunc ex-  
plicabimus.

Parallelogrammum ABDE circa decussatas lineas BH, EF  
ad rectos angulos constitue, & sit quantitas lunulae datæ  
BAED, ex alia crucis parte HIEF, elongeturq. parallela  
IH, quousque coeat cum lineâ AD in puncto G, mox lineâ  
GE discindens angulum DGH, ducatur per coniunctum B,  
quousque intercidat lineam AB, & ibi appone C, & ab C, pa-  
rallela ducatur CL, vsque donèc lineam GHI tetigerit in L,  
dico parallelogrammum FLI esse quantitatem trianguli BAC,  
qua superatur à parallelogrammo FH, quoniam parallelo-  
grammum BD est æquale parallelogrammo EL, deimpto pa-  
rallelogrammo FH, residuum erit FL, quod quaerimus.

Circu-

Circulum quadrato proximum constituere.

Prop. 16.

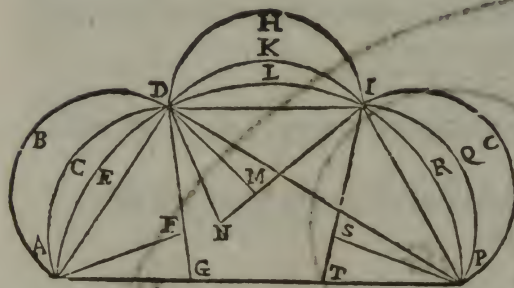


**A**bsoluta lunularum tractatione venio tandem ad circuli quadrationem ex ingenij facultate, sed initio circuli quadrationi approximabimur. Estoque sexto decupli circuli quadrans  $ABC$ , & ex semidiametri dimidio  $DE$ , sub sexto decuplus circulus constitutatur  $GH$ , amputetur pars quadrantis  $ADC$ , sitq.  $DBA$ , dico triangulum  $DBA$  circulo mutua parilitate constare secetur arcus quadrantis  $BA$  bifariam in  $F$ , connectanturq. recte  $BF$ ,  $FA$ , & à circulo  $GH$  itidem duæ illæ portiones quadrantis  $BF$ ,  $FA$ , dico triângula  $FDB$ ,  $ADF$  esse æqualia circulo, demptis duobus portionibus  $GH$ , æquales illis  $BF$ ,  $FA$  interclusis: quod patet ex constructione.

M Da-



## Datum circulum quadrare. Prop. 17.

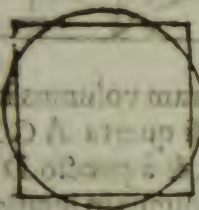
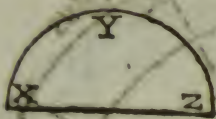


**E**Sto expositus circulus ADIP, inuariataq. circini apertura signetur in circuli circumferentia tres abscissus tres. n. recipiet compares, & coæquales sibi correspondentes semidiametro semicirculos (cogēte ad id Euclidis

Propos. 15.4.) quibus applicentur diametri AD, DI, IP, supra quorum centra incurventur semicirculi ABD, DHL, IOP, excurret linea ex D ad P, diuidaturq. angulus ADP per lineam DFG, & ex arte superius repetita fiant lunulæ ABDC, DHK, IOPQ, & triangula ADF, DMI, ISP cum suis subtriangulis suis sublunulis correspondentibus AFG, DNM, PST.

Quoniam semicirculus ALP constat quatuor semicirculis æqualibus semidiametro, si tollantur tres portiones communes AED, DLI, IRP, & tria triangula æqualia lunulis AFD, DMI, ISP, tollanturq. tres sublunulæ tribus subtriangulis respondentes ACDE, DKIL, IOPR, cum suis subtriangulis respondentibus illis AFG, NDM, SPT, vacuum reliquum intercedens rectilineum, vel trapezium GDNMIST valet quantum semicirculus quartus relictus XYZ, hoc inane valet semicirculus XYZ. Absoluamus igitur circulum cum suo quadrato valente trapezium illud, & quadratus erit circulus.

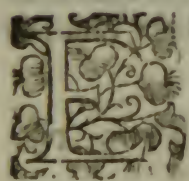
Nunc



Nunc evertatur, & faces-  
sat Hyppocratis Chij falla-  
cia circulum quadrare sata-  
gentis, quod putarat quem-  
admodum lunulæ dupli, &  
subdupli in quadratum ad-  
ducebantur, ita quamcumq.  
circularum cum suis rectili-  
neis æquationem; sed eius  
corrui demonstratio, nam  
res se aliter habet: quod  
enim est singulare in circulis  
se in dupla proportionem ex-  
crescentibus, dissentaneum  
est idem in reliquis existi-  
mare. Nos (ni fallimur) ex  
inventionem trianguli AFG  
sub lunula æreæ ADC.E respondentis, assecuti sumus.

Data portione nota, alteram cuiusq. propor-  
tionis sibi comparem peruestigare.

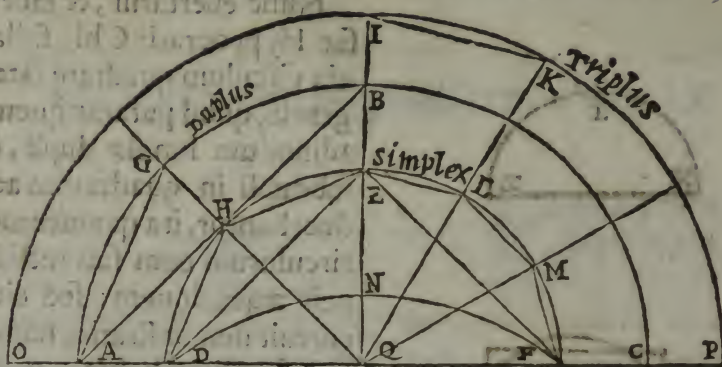
Prop. 18.



Sto linea. QP futura basis variorum circulo-  
rum, & semicirculus OIP triplus, ABC du-  
plus, & DEF simplex, siue subduplus, & sic  
de alijs alter supra alterum in quæsitâ ratio-  
ne semper excrescens, & à puncto Q, quod  
medium diametri possidet angulis vtriusque aequalibus ascen-  
dat linea QI semicirculus bifaria diuisione descendens: mox  
ex centro Q circumferentiam AB aqualiter partiens vsque  
ad C, transmittatur, ex altera parte dux alia circumferentiâ  
IP, trifariam secantes coæquantur, & sint QK, QM, & sit

M 2 data





data simplicis circuli portio DE, alteram volumus in dupla proportionem æqualem inuenire. Dupli quarta AGB discindatur bifariam, trahaturq. linea AG, & à puncto D ad E dupli portio subpingatur. Quoniam vidimus in lunula DEFN, portionem dupli quartæ circuli DNF valere duas quartas circuli subdupli, vel simplicis DHE, ELF, sed portio dupli DNF est æqualis AGB, quia eadem est dupli quarta. Ergo area conclusa in portione dupli DNF valet duas quartas subdupli DHE, ELF, & portio octaua dupli AG valet duas octauas subdupli DH, HE. Idem dicendum de tripla, nam circulus OIKP est triplus subtripli, vel simplicis circuli DHELF, & par sexta semicirculi tripli IK, valet tres sextas subtripli circuli EL, LM, MF, & sic de alijs cuiuscunque quantitaris, & incognita mensuræ dicendum: nam semper rata, & iusta proportio erit.

Ex portionibus circulum quadrare. Prop. 19.

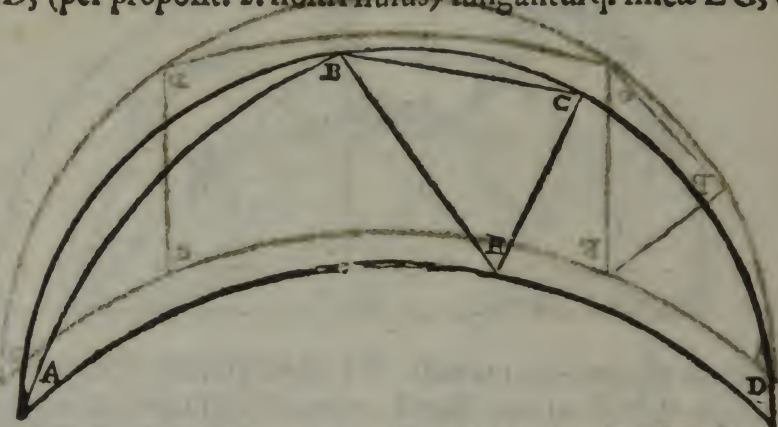
**E**X ea, quam modo exposuimus propositione dependet hæc portionum quadratio, quam ob oculos exponemus.

Esto proposita lunula ABCDE, ex qua (docente id 15. propos. nostri ante præteritam) minor lunula abscindatur, quæ





trames ducatur. mox lineæ ED compar subdupli reperiatur CD, (per proposit. 2. nostri huius) iunganturq. lineæ EC, &



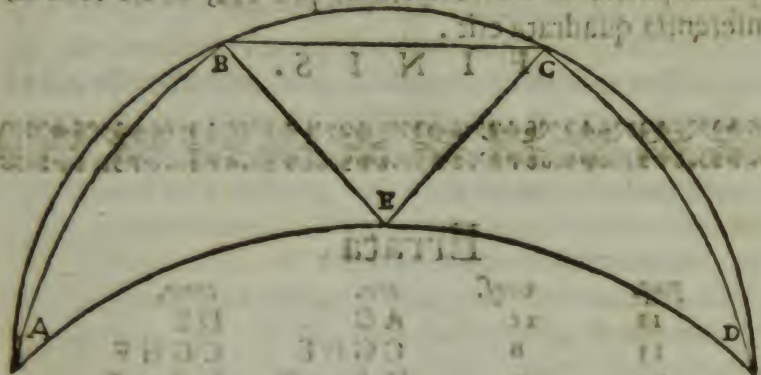
BC, recta lineam etiam connectatur. Quoniam AB, AE æquales, & eadem circumferentiæ sunt, si lineæ subtendantur arcibus AB, AE amputata portione AB, reposita suo loco AE notum erit, triangulum semicurvilineum ABE, lunula AB nota est (ex proposit. 15. presentis nostri) ergo lunulæ pars ABE nota erit, deme AB integræ, innotescet residuum BCDE lineæ ED comparem reddidimus CD. Ergo corniculus ECD notus erit, deme à toto residuo BCDE innotescet trimetrum semicurvilineum BEC, appinge lineam BC à terminis BC, & triangulum rectilineum BCE notum erit, subducto à curvilineo, & portio BC nota erit.

### Altera.

**V**el in lunula ABCDE à puncto A vsque ad E, nota in subduplo, & sit AB, & ex altera parte CD, & à puncto A, vsque ad B appinge lineam dupli AB, & ex altera parte CD, mox connecte lineas rectas BE, EC, BC, quia AB, AE æquales sunt, sic CD, DE, ergo lunulæ AB, CD etiam notæ, tolle, quia remanet par medij BEC nota, à noto

to fentitur lineo BGE, tolle rectilineum BOE; remanet no-

ta portio BC, fic & alie diuifiones imaginari poffunt, & por-



ta portio BC, fic & alie diuifiones imaginari poffunt, & por-  
tiones quadrari.

Data vna portione nota, totam circumferen-  
tiam circuli quadrare.

Prop. 20.

**S**It propositus circulus

ABCDEFGH, & fit  
portio nota AB, nos hanc  
circumferentiam metie-  
mur fepties AB, BC, CD,  
DE, EF, FG, GH, supererit  
HA, qua metiemur bis  
AB idest AI, IK, & super-  
erit KB, quæ portione n.  
IK ter menfurabit. Ergo  
AB fepties partiemur, qui-  
bus abfciffionibus fuas par-  
tes addemus. A portione igitur AB subducemus octogo-



num



num rectilineum, reliquum in septem portiones diuidemus;  
ex quibus portiones tres recipiemus pro HI, & sic tota cir-  
cumferentia quadrata erit.

F I N I S.

Errata.

pag.	vers.	err.	corr.
11	21	AC	DE
13	8	CGHE	CGHF
20	9	D, & ex D	L, & ex L
23	1	utrunque	utrunque
23	18	AFG	AFC
23	21	FG	FB
30	1	AD	AB
32	3	hic	hic
48	15	Prop 8.	Prop 7.
49	17	repositisq.	repositisq.
58	7	MNE	MNP
64	20	trapezium	trapezium
65	8	duplis	duplis
68	15	æquidistant	æquidistant
69	1	dimidium	quarta pars
70	12	descriptum	descriptas.
71	15	dempla	dempta
87	13	Semicuruilieum	Semicuruilineum

R O M A E,

Apud Bartholomæum Zannettum. M. DC. X.

SVPERIORVM PERMISSV.

## Typographus amico Lectori.

**I**oannis Baptista Porta V. Cl. ingenium Babylonis pal-  
mis confimile semper existimaui, ex illis enim mella con-  
ficere, cibos parare, vina colligere, contexere vestes, & sex-  
centa alia ad vitam vel sustinendam, vel ornandam sibi co-  
parare dicuntur Assyrij. En tibi, amice Lector fecundum  
ingenium Porta infinita, vel ornamenta, vel adiumenta par-  
turi, ac elaborauit. Ad excolendum animum philosophi-  
cas disputationes, ac mathematicas lucubrationes; ad re-  
creandum reficiendumq. Villam, Pomarium, & lepidissimas  
Comedias. Ad exornandum Admiranda, & alia multiplicis  
eruditionis volumina. Vno verbo nihil est in naturæ maie-  
state repositum, Nihil in huius vniuersi luce versatur, quod  
tibi Porta non suppediet. Plerisque iam olim frui contigit,  
multa propediem expecta, quæ nobis omni disciplinarum  
genere excultus, ac dignus longiore felicioreq. tuo Comes  
Anastasijs Lynceus, & Porta ipsi, quo cum pluri-  
ma de litteris contulit, pernecessarius, amantissime imperti-  
uit. Optandum interea est. vt Porta diutius sibi, tibi, Rei-  
publice viuat. Vt autem vno oculorum aspectu omnes ma-  
gni viri lucubrationes agnoscas illorum Catalogum subtexe-  
re visum est.

### *In lucem iam edita.*

Physiognomonica Humana tum Latina, tum Italica lingua.  
Physiognomonica Coelestis, libri sex, Lat.  
Phytognomonica, libri octo, Lat.  
Magia naturalis Lat. & Ital. primum quatuor libris, demum  
viginti absoluta.  
De Furtiuis litterarum notis vulgus de ziferis. libri quatuor,  
primum euulgati mox alio superacti.

Nomen Villa



Villa Lat. Pomarium, & Oliuetum olim seorsim, demum vno  
 volumine libris duodecim comprehensa. Lat.  
 De refractione optices, libri nouem, Lat.  
 De Curuilineis, libri duo primum, cui additus tertius liber  
 de Quadratura Circuli. Lat.  
 Interpretatio primi Almagesti cum Comm. Theonis Lat.  
 De munitione, libri tres, Lat.  
 Pneumaticorum, libri tres, Lat. Italicè spiritali: cioè d'inal-  
 zar. acque per forza d'aria.  
 De transmutationibus aeris, libri quatuor, Lat.  
 De Distillatione, libri nouem, Lat.  
 Ars reminiscendi, Lat. & Ital.

*Nondum edita.*

Catoptrica.  
 Theologumena, siue de numeris.  
 Taumatologia.  
 Scientiarum omnium Synopsis.

*Comedie stampate.*

La Fantescia.	I due Fratelli riuali.
L'Olimpia.	La Sorella.
La Cintia.	Il Moro.
La Turca.	La Trappolaria.
La Furla.	La Carbonaria.
L'Astrologo.	La Chiappinaria.
La Penelope.	Tragicomedia.

*Da stamparsi.*

Arte da Comporre Comedie.  
 Plauto tradotto.

S. Gior-

S. Giorgio. }  
 S. Dorotea. } Tragedie.  
 S. Eugenia. }

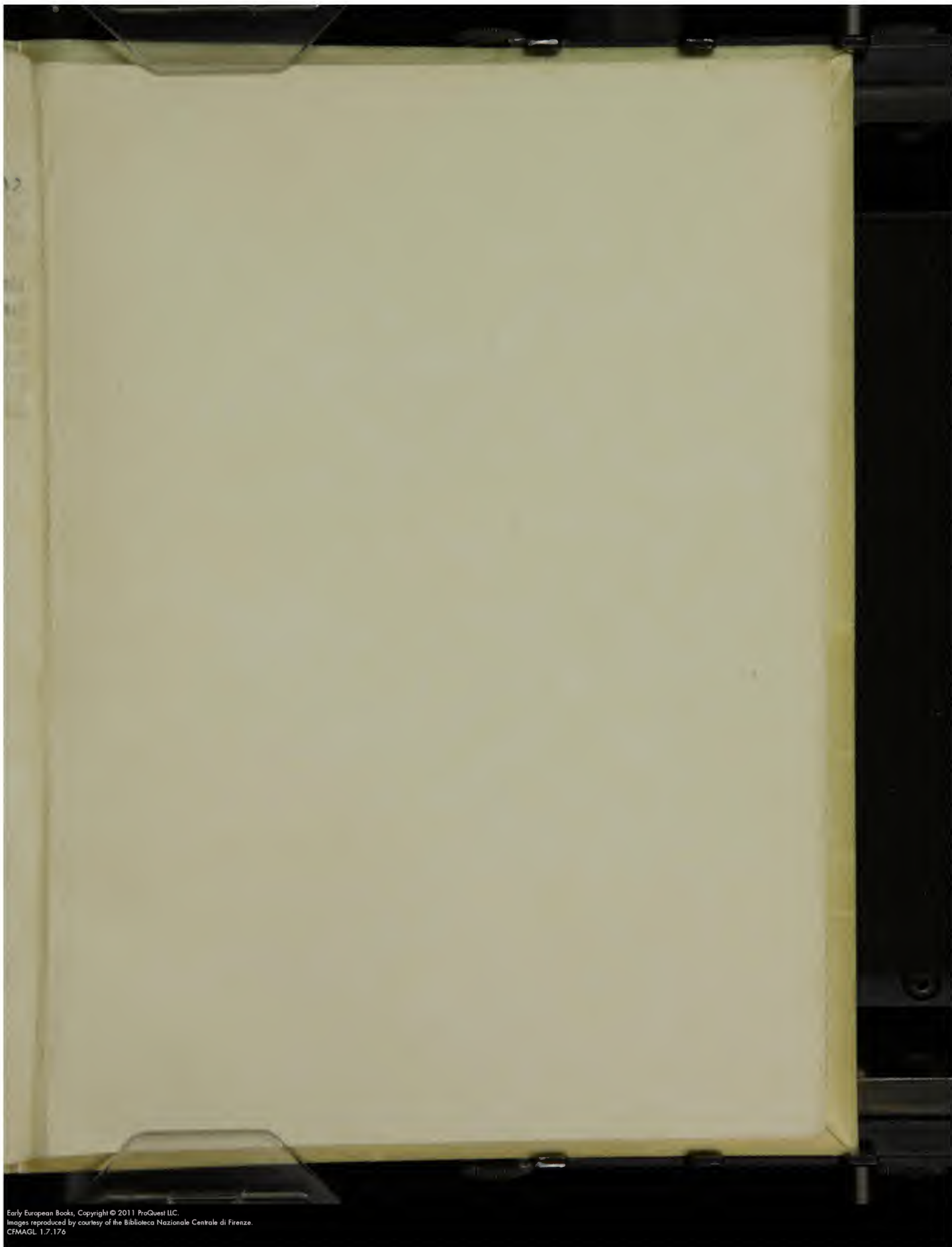
I simili. }  
 La notte. }  
 Il fallito. } Comedie.  
 La Strega. }  
 L'Alchimista. }  
 La Bufalaria. }

Cinque Comedie d'vna fauola sola con le medesime Persone, e la prima è argomento di se, & di tutte; la seconda è protesi di se & di tutte, con la peripatia per se, e tutte; la quinta è la Catastrofe per se, & tutte insieme.

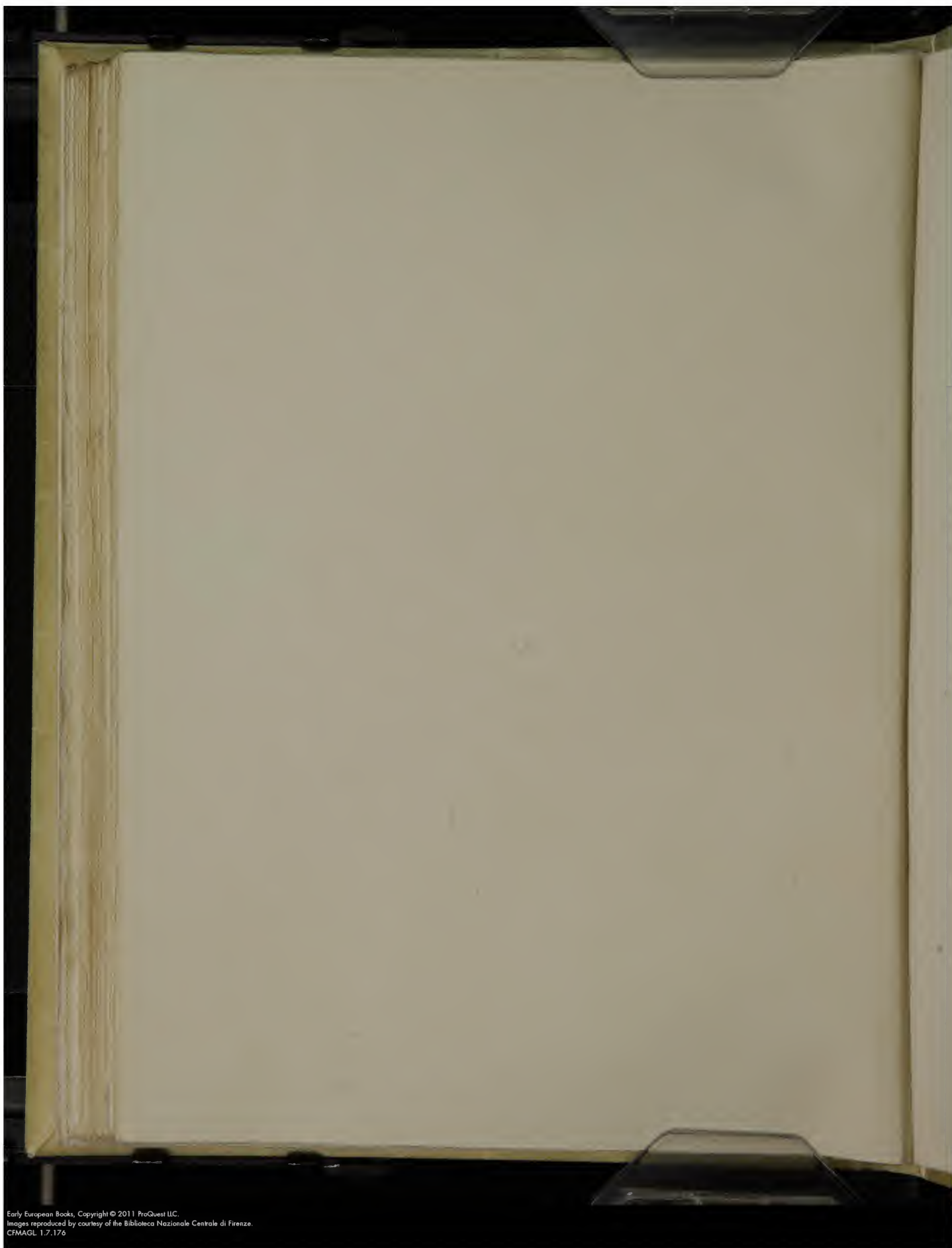
Due Comedie d'vna medesima fauola che l'vna si recita in Villa, e l'altra nella Città; e l'vna è intermedio dell'altra, voltandosi la Scena per ogn'atto, l'vna della Città, l'altra della Villa.

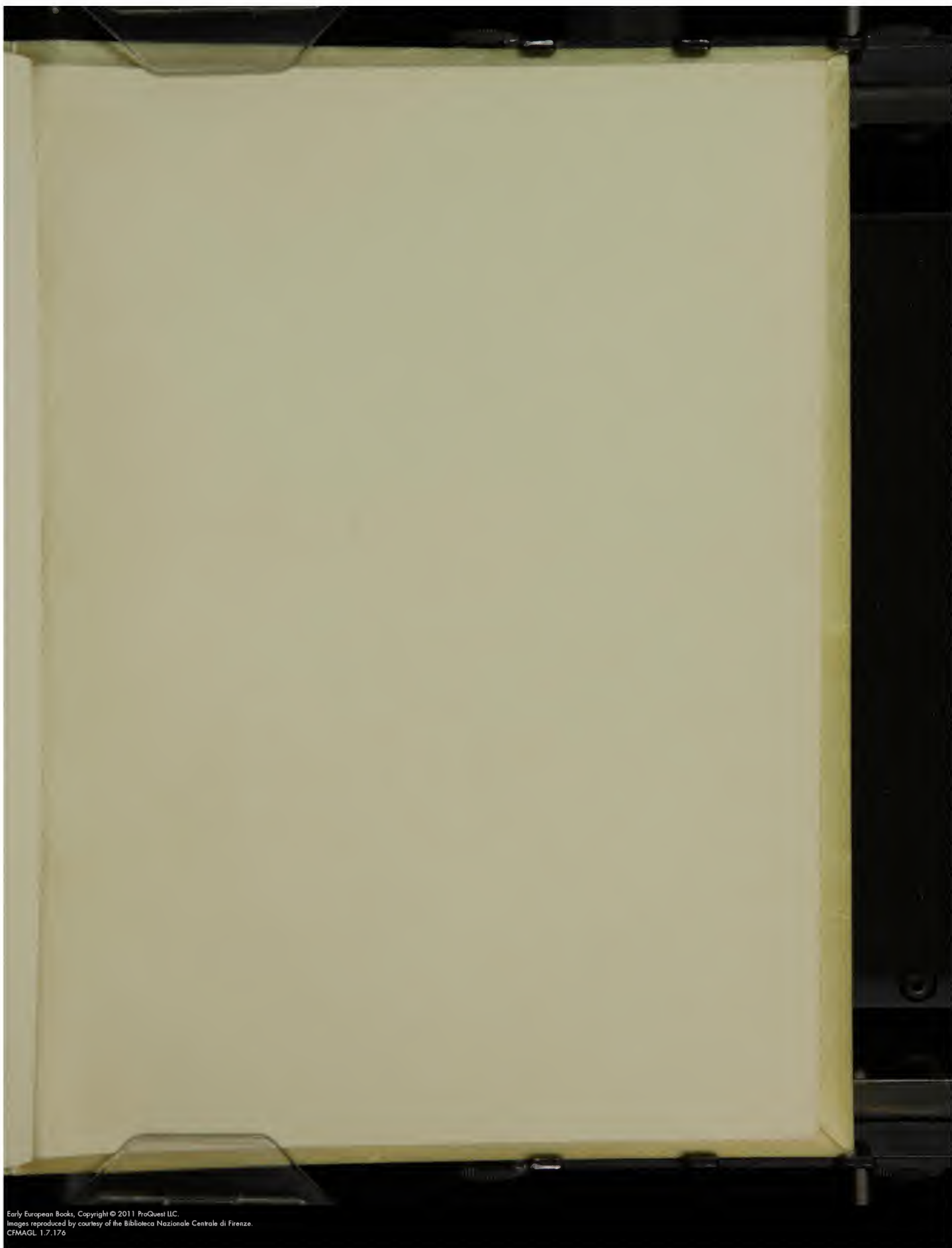


1. The first part of the book is a history of the  
 2. second part is a history of the  
 3. third part is a history of the  
 4. fourth part is a history of the  
 5. fifth part is a history of the  
 6. sixth part is a history of the  
 7. seventh part is a history of the  
 8. eighth part is a history of the  
 9. ninth part is a history of the  
 10. tenth part is a history of the

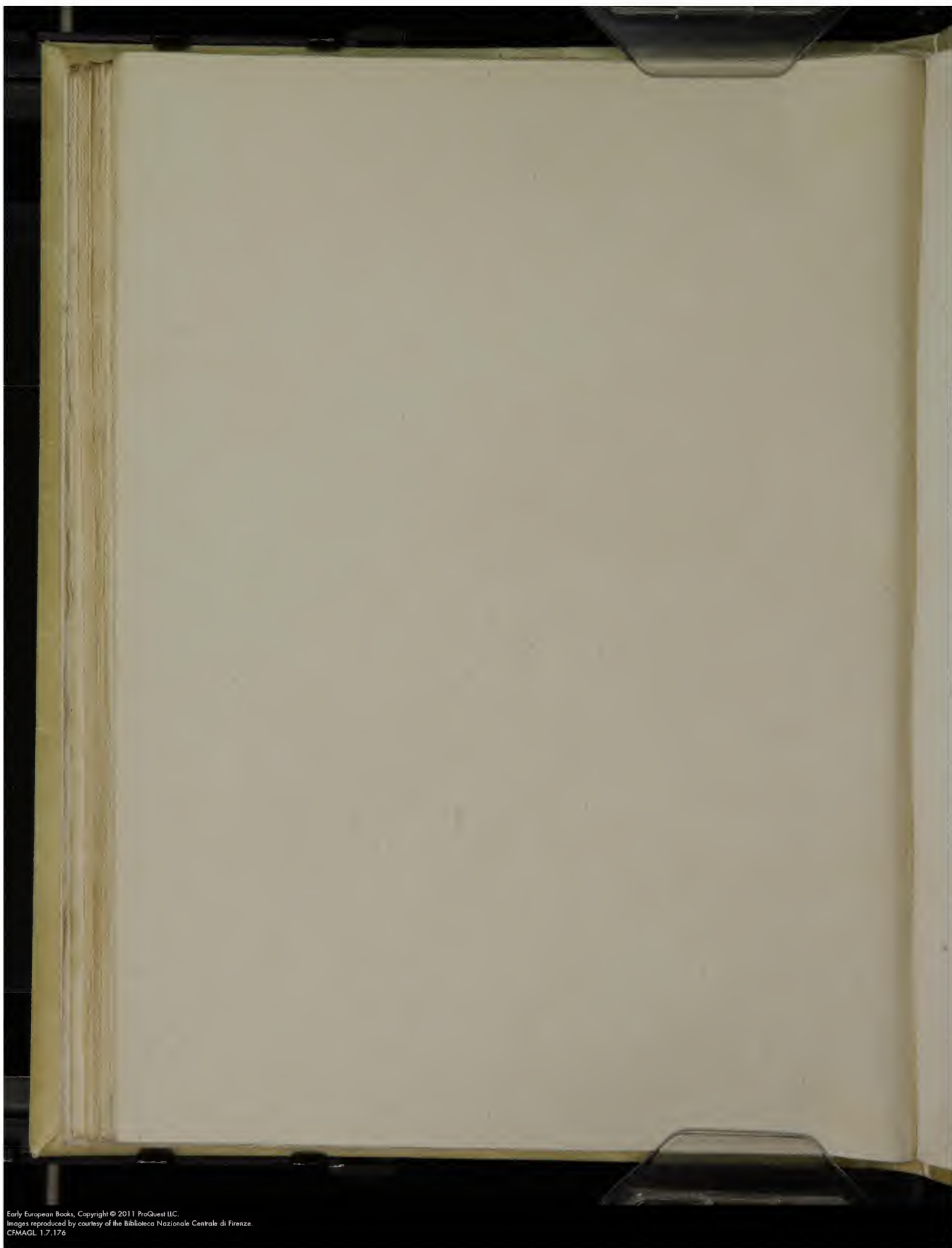












005644.684



